HIGH ORDER TYPE-2 TAKAGI-SUGENO-KANG FUZZY LOGIC SYSTEM

QUN REN

DÉPARTEMENT DE GÉNIE MÉCANIQUE

ÉCOLE POLYTECHNIQUE DE MONTRÉAL

PROPOSITION DE RECHERCHE DE DOCTORAT RAPPORT

PRÉSENTÉ EN VUE DE L’EXAMIN DE SYNTHÈSE ORAL

(GÉNIE MÉCANIQUE)

JUIN 2007

© QUN REN, 2007
# TABLE OF CONTENTS

<table>
<thead>
<tr>
<th>Section</th>
<th>Page</th>
</tr>
</thead>
<tbody>
<tr>
<td>TABLE OF CONTENTS</td>
<td>2</td>
</tr>
<tr>
<td>ABSTRACT</td>
<td>4</td>
</tr>
<tr>
<td>SECTION 1</td>
<td>5</td>
</tr>
<tr>
<td>BASIC TERMINOLOGY</td>
<td>5</td>
</tr>
<tr>
<td>1.1 Type-1 TSK Fuzzy Logic System</td>
<td>5</td>
</tr>
<tr>
<td>1.2 Type-2 TSK Fuzzy Logic and Fuzzy Logic System</td>
<td>7</td>
</tr>
<tr>
<td>1.2.1 Type-2 Fuzzy Logic</td>
<td>7</td>
</tr>
<tr>
<td>1.2.2 Type-2 TSK Fuzzy Logic System</td>
<td>9</td>
</tr>
<tr>
<td>1.2.3 Type-2 TSK Fuzzy Logic System inference engine</td>
<td>10</td>
</tr>
<tr>
<td>1.3 Comparison between type-1 and type-2 TSK Fuzzy Logic System</td>
<td>15</td>
</tr>
<tr>
<td>SECTION 2</td>
<td>18</td>
</tr>
<tr>
<td>LITERATURE REVIEW</td>
<td>18</td>
</tr>
<tr>
<td>2.1 Type-1 TSK Fuzzy Logic System</td>
<td>18</td>
</tr>
<tr>
<td>2.2 Type-2 Fuzzy Logic and Fuzzy Logic Systems</td>
<td>21</td>
</tr>
<tr>
<td>2.2.1 Type-2 Fuzzy set and Fuzzy Logic</td>
<td>21</td>
</tr>
<tr>
<td>2.2.2 Type-2 TSK Fuzzy Logic systems</td>
<td>22</td>
</tr>
<tr>
<td>2.3 Our Previous study</td>
<td>23</td>
</tr>
<tr>
<td>2.4 Summary of Literature Review</td>
<td>26</td>
</tr>
</tbody>
</table>
SECTION 3 ............................................................................................................................. 27

RESEARCH METHODOLOGY ............................................................................................ 27

3.1 Problematic .................................................................................................................. 27

3.2 Research Objectives .................................................................................................. 28

3.3 Research Methodology .............................................................................................. 28

3.3.1 High order type-2 TSK Fuzzy Logic System ......................................................... 28
3.3.2 Sensitivity analysis for type-2 TSK Fuzzy Logic System ..................................... 30
3.3.3 Application ............................................................................................................. 30

SECTION 4 ............................................................................................................................ 32

CONCLUSION .................................................................................................................... 32

4.1 Expected Contributions ............................................................................................. 32
4.2 Research Schedule ..................................................................................................... 32

REFERENCES .................................................................................................................... 34
ABSTRACT

This research proposal considers the problem of first order type-2 Takagi-Sugeno-Kang (TSK) Fuzzy Logic System (FLS) identification using subtractive clustering: the curse of dimensionality. The background information and literature survey concerning this subject are first given. The research objective is proposing the architecture, inference engine and a design method of high order type-2 TSK FLS. Methodologies to achieve the research objective are analyzed and formulated. Applications in mechanical engineering are used to demonstrate the advantages of type-2 FLSs: higher accuracy result and its unique interval set of output which can be used to analyze the uncertainties associated with the experimental system. In addition, the future research is scheduled in a three year project with detailed steps.
SECTION 1

BASIC TERMINOLOGY

This section describes basic conception of type-1 TSK FLS, some definition of type-2 fuzzy logic, structure of first order type-2 TSK FLS.

1.1 Type-1 TSK Fuzzy Logic System

The generalized $k$th rule of $m$-order type-1 TSK Multi-Input Single-Output (MISO) model can be described by the following expression [1]:

\[
\text{IF } \begin{array}{l}
\text{x}_1 \text{ is } Q_1^k \\
\text{and } \text{x}_2 \text{ is } Q_2^k \\
\text{and } \ldots \text{ and } \text{x}_n \text{ is } Q_n^k,
\end{array}
\]

\[
\text{THEN } Z \text{ is } W^k = \sum_{i=0}^{m} \sum_{q=0}^{m} \sum_{j=0}^{m} p_{j_1i_1q_1} \cdot x_1^{i_1} x_2^{j_1} \cdots x_n^{q_1}
\]

where $p_{j_1i_1q_1}$ represents $m$-order coefficients for $j = 1, 2, \ldots, m, i = 1, 2, \ldots, m, \ldots, q = 1, 2, \ldots, m$ in the $k$th rule where $j, i, \ldots, q$ are the order of variables.

When $m = 1$, eq.(1.1) become a first order type-1 model. A generalized type-1 TSK model can be described by fuzzy IF-THEN rules which represent input-output relations of a system. For a type-1 TSK model, its $k$th rule can be expressed as:
IF \( x_1 \) is \( Q_1^k \) and \( x_2 \) is \( Q_2^k \) and \( \ldots \) and \( x_n \) is \( Q_n^k \),

THEN \( Z \) is \( w^k \) ? \( f^k (x_1, x_2, \ldots, x_n) ? p_0^k - p_1^k x_1 - p_2^k x_2 - \ldots - p_n^k x_n \) \hfill (1.2)

where \( x_1, x_2 \ldots x_n \) and \( Z \) are linguistic variables; \( Q_1^k, Q_2^k, \ldots, \) and \( Q_n^k \) are type-1 fuzzy sets on universe of discourses \( X_1, X_2, \ldots, X_n \); \( p_0^k, p_1^k, p_2^k, \ldots, p_n^k \) are constant regression parameters.

Because Gaussian basis functions (GBFs) have the best approximation property \[2\], Gaussian functions are usually chosen as the MFs. A type-1 Gaussian MF can express by using formula for the \( v \)th variable:

\[
Q_v^k = \exp \left[ -\frac{1}{2} \left( \frac{x_v / x_v^k}{\nu} \right)^2 \right]
\]

\hfill (1.3)

where \( x_v^k \) is the mean of the \( v \)th input feature in the \( k \)th rule for \( v \in [0, n] \), \( \nu \) is the standard deviation of Gaussian MF.

When certain input values \( x_1^0, x_2^0, \ldots, x_n^0 \) are given to \( x_1, x_2, \ldots, x_n \), the conclusion from a TSK rule \( r_k \) is a crisp value \( w_k^* \):

\[
w_k^* ? f^k (x_1^0, x_2^0, \ldots, x_n^0) \]

\hfill (1.4)
having some rule firing strength (weight) defined as

\[ \chi_k = o_1^k (x_1^0) \cap o_2^k (x_2^0) \cap \ldots \cap o_n^k (x_n^0) \]  

\[ (1.5) \]

\( \chi_k \) is the activation value of weight for the antecedent of the rule \( r_k \). Moreover, \( o_1^k (x_1^0), o_2^k (x_2^0), \ldots, o_n^k (x_n^0) \) are membership grade for fuzzy sets \( Q_1^k, Q_2^k, \ldots, \) and \( Q_n^k \) in the rule \( r_k \). The symbol \( \cap \) is a conjunction operator, which is a T-norm. In this thesis, the conjunction operator is the minimum operator \( \land \) or the product operator \( \ast \).

The output of a TSK fuzzy system with \( m \) rules can be expressed (using weighted average aggregation) as

\[ w^* = \frac{\sum_{k=1}^{m} \chi_k w_k^*}{\sum_{k=1}^{m} \chi_k} \]  

\[ (1.6) \]

1.2 Type-2 TSK Fuzzy Logic and Fuzzy Logic System

1.2.1 Type-2 Fuzzy Logic

A FLS that is described completely in terms of type-1 fuzzy sets is called a type-1 FLS, whereas a FLS that is described using at least one type-2 fuzzy set is called a type-2 FLS. Type-1 FLSs cannot directly handle rule uncertainties because they use type-1 fuzzy sets that are certain. Type-2 FLSs, on the other hand, are very useful in circumstances in which it
is difficult to determine an exact membership function for a fuzzy set; hence, they can be used to handle rule uncertainties and even measurement uncertainties. Type-2 FLSs have been developed that satisfy the following fundamental design requirement:

**When all sources of uncertainty disappear, a type-2 FLS must reduce to a comparable type-1 FLS.**

Karnik and Mendel [3] provide the following definition of a type-2 fuzzy set:

“A type-2 fuzzy set is characterized by a fuzzy membership function, i.e. the membership value (or membership grade) for each element of this set is a fuzzy set in [0, 1], unlike a type-1 fuzzy set where the membership grade is a crisp number in [0, 1].”

The characterization in this definition of type-2 fuzzy sets uses the notion that type-1 fuzzy sets can be thought of as a first order approximation to uncertainty and, therefore, type-2 fuzzy sets provides a second order approximation. They play an important role in modeling uncertainties that exist in fuzzy logic systems [4], and are becoming increasingly important in the goal of “Computing with Words” [5] and “Computational Theory of Perceptions” [6].

An example of a type-2 principal MF is the Gaussian MF depicted in Figure 1.1, whose vertices have been assumed to vary over some interval of value. The footprint of uncertainty (FOU) associated with this type-2 MF is a bounded shaded region, in Figure 1.1. FOU represents the entire shaded interval type-2 fuzzy set \( \tilde{Q} \). Upper MF and Lower MF are two
type-1 MFs that are bounds for the FOU of a type-2 set $\tilde{Q}$. The intersections of the crisp input $x^0$ with the lower MF is the degree $\tilde{Q}$ and with the upper MF is the degree $\tilde{Q}$.

![Figure 1.1 Type-2 Gaussian MF](image)

The type-2 FL theory is presented in Mendel’s book “Uncertain Rule-Based Fuzzy Logic Systems – Introduction and New Directions” [7].

### 1.2.2 Type-2 TSK Fuzzy Logic System

A generalized $k$th rule in the first-order type-2 TSK fuzzy MISO system [7] with a rule base of $m$ rules, each having $n$ antecedents, the rule $r_k$ can be expressed as eq. (1.7) instead of eq. (1.2) in type-1 TSK fuzzy MISO system, i.e.,
IF $X_1$ is $\tilde{Q}_1$ and $X_2$ is $\tilde{Q}_2$ and ... and $X_n$ is $\tilde{Q}_n$, 

THEN $Z$ is $w^k$ if $p_0 - p_1 x_1 - p_2 x_2 - ... - p_n x_n$ (1.7)

where $p_0, p_1, ..., p_n$ are consequent type-1 fuzzy sets, while $w^k$ is the consequent of the $k$th IF-THEN rule. Moreover, $\tilde{Q}_1, \tilde{Q}_2, ..., \tilde{Q}_n$ are fuzzy sets on universe of discourses $X_1, X_2, ..., X_n$. $k \in [1, m]$, where $m$ is the total number of rules.

A type-2 Gaussian MF can express by using formula for the $v$th variable:

$$
\tilde{Q}_v \sim \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^k}{\sigma^k_v} \right)^2 \right] \ (1.8)
$$

where $a^k_v$ is spread percentage of mean $x_v^k$ as shown in Figure 1.2, $\sigma^k_v$ is the standard deviation of Gaussian MF of the $k$th rule.

1.2.3 Type-2 TSK Fuzzy Logic System inference engine

For the most general model of type-2 TSK FLS, antecedents are type-2 fuzzy sets and consequents are type-1 fuzzy sets, then consequent parameter $p_0, p_1, ..., p_n$ are assumed as convex and normal type-1 fuzzy number subsets of the real line, so that they are fuzzy number., i.e.
\[
\mu_{k}^j \left[ \begin{array}{cccc}
    c_j & s_j & c_j - s_j & s_j \\
\end{array} \right]
\]  

(1.9)

where \( c_j \) denotes the centre (mean) of \( p_j \), and \( s_j \) denotes the spread of fuzzy number \( p_j \).

\( j \in \{0, n\} \), where \( j \) is the total number of rules.

Similar to that shown in Figure 1.1, MF degree \( \tilde{O}_1^k, \tilde{O}_2^k, \ldots, \tilde{O}_n^k \) are interval set,

\[
\tilde{O}_1^k = \left[ \begin{array}{c}
    O_1^k, O_1^k \\
\end{array} \right] \quad k \in \{1, \ldots, m\}
\]

(1.10)
Mendel in his book [7] describes the inference computation of type-2 TSK FLS which is similar to a type-1 TSK FLS that no defuzzification is needed.

The firing set [7] alters the consequent set for a fired rule in a singleton type-2 FLS. It conveys the uncertainties of the antecedents to the consequent set. The total firing set $f^{k}$ for the rule $r_k$ is interval type-1 set, i.e. $f^{k} = [f^{k}, \tilde{f}^{k}]$. The explicit dependence of $f^{k}$ can be computed as:

$$f^{k} \leftarrow o^{k}_{i} (x_{1}) \cap o^{k}_{2} (x_{2}) \cap ... \cap o^{k}_{n} (x_{n})$$ \hspace{2cm} (1.11)

$$\tilde{f}^{k} \leftarrow \tilde{o}^{k}_{i} (x_{1}) \cap \tilde{o}^{k}_{2} (x_{2}) \cap ... \cap \tilde{o}^{k}_{n} (x_{n})$$ \hspace{2cm} (1.12)

The consequent $w^{k}$ of the rule $r_k$ is a type-1 fuzzy set because it is a linear combination of type-1 fuzzy sets [7]. It is also an interval set, i.e.,

$$w^{k} \leftarrow [w^{k}_{l}, w^{k}_{r}]$$ \hspace{2cm} (1.13)

where $w^{k}_{l}$ and $w^{k}_{r}$ are its two end-points, while $w^{k}_{l}$ is the consequent of a type-1 TSK FLS, whose antecedent MF are the lower MFs of the type-2 TSK FLS. Moreover, $w^{k}_{r}$ is the consequent of a type-1 TSK FLS whose antecedent MF are the upper MFs of the type-2
TSK FLS and $w_i^k$ and $w_r^k$ can be computed as

$$w_i^k = \sum_{i=1}^{n} c_i^k x_i - c_0^k / \sum_{i=1}^{n} s_i^k |x_i| - s_0^k$$  \hspace{1cm} (1.14)$$
$$w_r^k = \sum_{i=1}^{n} c_i^k x_i - c_0^k - \sum_{i=1}^{n} s_i^k |x_i| - s_0^k$$  \hspace{1cm} (1.15)$$

$\tilde{w}$ is the extended output of a type-2 TSK FLS. It reveals the uncertainty of the output of a type-2 TSK FLS due to antecedent or consequent parameter uncertainties. The interval set of total output $\tilde{w}$ for $m$ rules of the system (1.7) is obtained by applying the Extension Principle [8, 9]. When interval type-2 fuzzy sets are used for the antecedents, and interval type-1 fuzzy sets are used for the consequent sets of a type-2 TSK rules [10-12], $\tilde{w}$ can be obtained by

$$\tilde{w} = [w_l, w_r]$$

$$= \left[ \prod_{i=1}^{m} w_i^l, w_r^l \right] \times \left[ \prod_{i=1}^{m} w_i^r, w_r^r \right] \geq \left[ \prod_{i=1}^{m} \int_{f_i^l}^{f_i^r} \int_{f_i^r}^{f_i^l} \right] \frac{1}{\sum_{k=1}^{m} f_k W_k}$$  \hspace{1cm} (1.16)$$

Hence $\tilde{w} = [w_l, w_r]$ is an interval type-1 set. To compute $\tilde{w}$, its two end-points $w_l$ and $w_r$ must be computed. Let the value of $f^k$ and $w^k$ that are associated with $w_l$ be denoted $f_i^k$ and $w_i^k$, respectively, and those associated with $w_r$ be denoted $f_r^k$ and
\( w_r^k \). The two endpoints \( w_l \) and \( w_r \) can be obtained as follows:

\[
\begin{align*}
    w_l &= \frac{\sum_{k=1}^m f_l^k w_l^k}{\sum_{k=1}^m f_l^k} \\
    w_r &= \frac{\sum_{k=1}^m f_r^k w_r^k}{\sum_{k=1}^m f_r^k}
\end{align*}
\]  

(1.17)  

(1.18)

To compute \( w_l \) and \( w_r \), \( f_l^k \) and \( f_r^k \) have to be determined. Moreover \( w_l \) and \( w_r \) can be obtained by using the iterative procedure proposed by Karnik and Mendel [13]. Here, the computation procedure is briefly provided.

Without loss of generality, assume that the pre-computed \( w_r^k \) are arranged in ascending order; \( w_r^1 \leq w_r^2 \leq \ldots \leq w_r^m \), then,

Step 1: Compute \( w_r \) in (1.18) by initially setting \( f_r^k = \frac{f_r^k + \bar{f}_r^k}{2} \) for \( k = 1, \ldots, m \), where

\( f_r^k \) and \( \bar{f}_r^k \) have been previously computed using (1.11, 1.12), respectively, and let

\( w_r^0 \equiv w_r \).

Step 2: Find \( R (1 \leq R \leq m / 1) \) such that \( w_r^R \leq w_r' \leq w_r^{R-1} \).
Step 3: Compute \( w_r \) in (1.18) with \( f_r^k = f_i^k \) for \( k \leq R \) and \( f_r^k = \tilde{f}_r^k \) for \( k \geq R \), and let \( w_r^\prime \equiv w_r \).

Step 4: If \( w_r^\prime \neq w_r^\prime \), then go to Step 5. If \( w_r^\prime = w_r^\prime \), then stop and set \( w_r^\prime \equiv w_r \).

Step 5: Set \( w_r^\prime = w_r^\prime \), and return to Step 2.

The procedure for computing \( w_l \) is very similar to the one just given for \( w_r \). Replace \( w_r^k \) by \( w_l^k \), and, in Step 2 find \( L \ (1 \leq L \leq m/1) \) such that \( w_l^L \leq w_l^L \leq w_l^{L-1} \).

Additionally, in Step 3, compute \( w_l \) in (1.17) with \( f_l^k = f_i^k \) for \( k \leq L \) and \( f_l^k = \tilde{f}_l^k \) for \( k \geq L \).

In an interval type-2 TSK FLS, output \( \tilde{w} \) is an interval type-1 fuzzy set, so the crisp output \( w^* \) of any interval type-2 TSL FLS can be obtained by using the average value of \( w_l \) and \( w_r \). Hence, the crisp output of type-2 TSK FLS is

\[
\frac{w_l - w_r}{2} \tag{1.19}
\]

1.3 Comparison between type-1 and type-2 TSK Fuzzy Logic System

Type-1 and type-2 TSK FLSs are characterized by IF-THEN rules and no defuzzification is needed in the inference engine, but they have different antecedent and
consequent structures. Assuming FLSs with \( m \) rules and \( n \) antecedents in each rule, a type-1 TSK FLS is compared with a type-2 TSK FLS in Table 1.1.

From Table 1.1, a type-2 TSK FLS has more design degrees of freedom than does a type-1 TSK FLS because its type-2 fuzzy sets are described by more parameters than type-1 fuzzy sets [7]. This suggests that a type-2 TSK FLS has the potential to outperform a type-1 TSK FLS because of its larger number of design degrees of freedom.
Table 11 Comparison between type-1 and type-2 TSK FLS

<table>
<thead>
<tr>
<th>TSK FLS</th>
<th>Type-1 TSK FLS</th>
<th>Type-2 TSK FLS</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>(A2-C1)</td>
</tr>
<tr>
<td></td>
<td>Antecedents</td>
<td>(A2-C0)</td>
</tr>
<tr>
<td></td>
<td>Type-1 fuzzy set</td>
<td>(A1-C1)</td>
</tr>
<tr>
<td></td>
<td>Consequent Parameters</td>
<td></td>
</tr>
<tr>
<td></td>
<td>Crisp number</td>
<td>Fuzzy number</td>
</tr>
<tr>
<td></td>
<td></td>
<td>Crisp number</td>
</tr>
<tr>
<td></td>
<td>Output</td>
<td>A interval set of output</td>
</tr>
<tr>
<td></td>
<td>A crisp point</td>
<td>A point output</td>
</tr>
<tr>
<td></td>
<td>Number of design parameters</td>
<td></td>
</tr>
<tr>
<td></td>
<td>(3p+1)M*</td>
<td>(5p+2)M*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4p+1)M*</td>
</tr>
<tr>
<td></td>
<td></td>
<td>(4p+2)M*</td>
</tr>
</tbody>
</table>
SECTION 2

LITERATURE REVIEW

Because the universal approximation property, TSK FLSs are widely developed and used. Type-2 fuzzy sets and type-2 fuzzy systems are becoming very well established and are gaining more and more in popularity. This section recalls the previous studies about type-1 and type-2 TSK FLSs, including our proposed subtractive clustering based type-2 TSK fuzzy system identification algorithm [16].

2.1 Type-1 TSK Fuzzy Logic System

The proposed linguistic approach by Zadeh [17, 18], following the first paper “Fuzzy Sets” in 1965 [19], has the capability to model complex system behavior in such a qualitative way that the model is effective and versatile in capturing the behavior of ill-defined systems with realistic approximations. Motivated by these ideas of “fuzzy algorithm” and “linguistic analysis”, Mamdani first applied fuzzy logic (FL) to control [20]. This topic is known as fuzzy algorithmic control or linguistic control. The main problem with fuzzy control is the design of a fuzzy controller where we usually take an expert-system-like approach. That is, we derive fuzzy control rules from the human operator’s experience and/or engineer’s knowledge, which are mostly based on their qualitative knowledge of an objective system. A set of fuzzy control rules is a linguistic model of human control actions which is not based on a mathematical description of human control actions but is directly based on a human way of thinking about plant operation.
Zadeh’s proposal of linguistic approach is effective and versatile in modeling ill-defined systems with fuzziness or fully defined systems with realistic approximations. Later it is expanded into fuzzy systems modeling as a qualitative modeling by Tong [21], Pedrycz [22], Takagi and Sugeno [23, 24], Trojan et al. [25], Sugeno and Kang [26], Sugeno and Tanaka [27], Sugeno and Yasukawa [28].

Takagi-Sugeno-Kang (TSK) type fuzzy model structure, also being referred to as TSK fuzzy logic systems (FLSs), after Takagi, Sugeno and Kang, has attracted many attentions. It was proposed in an effort to develop a systematic approach to generate fuzzy rules from a given input-output data set. This model consists of rules with fuzzy antecedents and mathematical function in the consequent part. Usually conclusion function is in form of dynamic linear equation [23, 24]. The antecedents divide the input space into a set of fuzzy regions, while consequents describe behaviours of the system in those regions. There is a need to develop a semi-automatic method to obtain those models based on sets of input-output data. The main difference with more traditional [20] fuzzy rules is that the consequent of the rules are a function of the input variables values. This approach has demonstrated to have a powerful representative capability, being able to describe non-linear mappings using a small number of simple rules.

In fuzzy system modeling, system identification can be done by using fuzzy clustering techniques. Clustering methods are proposed to identify natural grouping of data from a large data set such that a concise representation of system’s behavior is produced. Yager and Filev [29-31] developed the mountain method for estimating cluster centroids. Mamdani and Assilian [32], Bezdek [33] and Bezdek et al. [34] proposed a variety of clustering algorithms, including hierarchical, k-means, and fuzzy c-means algorithms. The initial
selection of cluster details has been made automatic by Emami et al. [35]. Chiu’ approach [14, 15] known as subtractive clustering reduce the computational complexities. In the literature, different modeling techniques can be found [36].

TSK FLSs are widely used for model-based control and model-based fault diagnosis. This is due to the model’s properties of, on one hand being a general nonlinear approximator that can approximate every continuous mapping, and on the other hand, being a piecewise linear model that is relatively easy to interpret [37] and whose linear submodels can be exploited for control and fault detection [38, 39].

Zero order and first order TSK FLSs based on clustering method suffer the curse of dimensionality – the number of rules increases exponentially with the number of input variables and the number of MFs per variable [40]. High order consequent TSK rules can reduce drastically the number of rules needed to perform the approximation, and improve transparency and interpretation in many high dimensional situations. High order TSK fuzzy systems were applied in numerous fields:

- noise cancellation [41];
- creating personalized models [42];
- diagnosis of tool wear [43];
- function approximation [44, 45];
- system identification [1];
- medical decision support [46];
- pattern recognition [47];
- fuzzy controller [48, 49];
• temperature control [50];
• machining process modeling [51].

2.2 Type-2 Fuzzy Logic and Fuzzy Logic Systems

2.2.1 Type-2 Fuzzy set and Fuzzy Logic

The original fuzzy logic (FL), founded by Zadeh [17], is unable to handle uncertainties. By "handle," that means "to model and minimize the effect of.". The expanded FL, type-2 FL, is able to handle uncertainties because it can model them and minimize their effects. And, if all uncertainties disappear, type-2 FL reduces to type-1 FL, in much the same way that, if randomness disappears, probability reduces to determinism [7].

Type-2 fuzzy sets were introduced by Zadeh in [9]. The main concept of type-2 fuzzy logic is that “words mean different things to different people”; thus, there are uncertainties associated words. Based on Extension Principle [9, 17], Algebraic structure of type-2 fuzzy sets were studied by Mizumoto and Tanaka [52, 53], Roger [54], also Niminen [55]. Debois and Prade [56-58] discussed fuzzy valued logic depended on minimum conjunction and gave a formula for the composition of type-2 relations. Karnik and Mendel [13] extended those works and obtained practical algorithms for conjunction, disjunction and complication operations of type-2 fuzzy sets, they also developed a general formula for the extended composition of type-2 relations which is consider as an extension of the type-1 composition. Based on this formula, Karnik et al [3] established a mathematical theory of type-2 Mamdani FLSs in 1999. Later, Liang and Mendel developed a more complete theory for interval type-2 FLSs [59, 60].
By using the discrete probability theory, Mendel and John [10-12] used embedded interval valued type-2 fuzzy sets to aid the discussion of interval valued type-2 fuzzy sets and to the join and meet of interval valued type-2 fuzzy sets, and made type-2 fuzzy sets easy to understand and explain.

Type-2 fuzzy sets are described by MFs that are characterized by more parameters that are MFs for type-1 fuzzy sets. Hence, type-2 fuzzy sets provide us with more design degrees of freedom; so using type-2 fuzzy sets has the potential to outperform using type-1 fuzzy sets, especially when we are in uncertain environment.

### 2.2.2 Type-2 TSK Fuzzy Logic systems

Type-2 TSK FLSs are presented in 1999 by Liang and Mendel [61], and type-2 TSK FLSs have the potential to be used in control and other areas where a type-1 TSK model may be unable to perform well because of its large numbers of design parameters. In 2001, Mendel published his famous book “Uncertain Rule-Based Fuzzy Logic Systems – Introduction and New Directions” [7] including the architecture, inference engine and design method of first-order interval singleton. First-order interval non-singleton type-2 TSK FLS was presented by Mendez and al [62, 63] in 2006.

There are several different design method design method for type-2 TSK FLS in the literature:
• Back-propagation method [7];
• Using hybrid learning algorithm [64];
• Using subtractive clustering [16].

We propose the last one --- subtractive clustering based type-2 TSK fuzzy system identification algorithm in 2006. This method directly extends a subtractive clustering based type-1 TSK FLS to its type-2 counterpart with emphasis on interval set. Comparing with the first two methods in which type-2 TSK model is directly formed from input-output data, our algorithm is easier to understand for type-1 TSK experts and easier to handle the precision of type-2 TSK model using subtractive clustering. This method will be described in detail in Section 2.3.

Here the applications that have appeared in the literature for type-2 TSK fuzzy systems are categorize as following:

• System modeling [61];
• Decision feedback equalizer for nonlinear time-varying channels [65, 66];
• Prediction of the transfer bar surface temperature at finishing scale breaker entry zone [62, 63];
• Function approximation [16];
• Identification of rigid-body dynamics of robotic manipulators [67].

2.3 Our Previous study
Based on type-2 TSK FLS theory [7], we proposed a design method of type-2 TSL FLS -- subtractive clustering based type-2 TSK fuzzy system identification algorithm [10]. This method directly extends a type-1 TSK FLS to its type-2 counterpart with emphasis on interval set. Type-1 Gaussian MFs is used as principle MFs to expand type-1 TSK model to type-2 TSK model. The proposed type-2 TSK modeling identification algorithm has following steps:

Step 1: Use Chiu’s subtractive clustering method combined with least squares estimation algorithm [14, 15] to pre-identify a type-1 fuzzy model from input/output data. The type-1 fuzzy model can be expressed by eq.(1.2)

Step 2: Calculate root-mean-square-error (RMSE) of the type-1 fuzzy model. If RMSE is bigger than expected error limitation, go to Step3. If not, end program, which means that the type-1 model is acceptable, and it is no need to use type-2 TSK model.

Step 3: Use type-1 Gaussian MFs as principle MFs to expand type-1 TSK model to type-2 TSK model:

- Spread cluster center to expand premise MFs from type-1 fuzzy sets to type-2 fuzzy sets.

\[
\frac{Q_k}{x_v} \exp \left\{ \frac{1}{2} \left( \frac{x_v - a_v}{\sigma_v} \right)^2 \right\}
\]

as depicted in Figure 1.1.

- The deviation for each rule varies from each other in the fuzzy model to get the best model. Equation (2.1) is changed to (2.2) as illustrated in Fig. 2.1 where
constant $\nu^k$ is replaced by $\nu^k_i$.

$$
\hat{Q}_{v} \approx \exp \left[ \frac{1}{2} \left( \frac{\chi_{v} / \chi_{v}^{k} \pm a_{v}^{k}}{\nu_{v}^{k}} \right)^{2} \right]
$$

(2.2)

Figure 2.1 Standard deviation of Gaussian MF

- Spread the parameters of consequence to expand consequent parameters in (1.2) from certain value to fuzzy numbers in (2.3).

$$
\hat{p}_{j} \approx p_{j}^{k}(1 \pm b_{j}^{k})
$$

(2.3)

where $b_{j}^{k}$ is the spread percentage of fuzzy numbers $p_{j}$. 
Step 4: By using Mendel proposed inference engine [7], compute the interval value of the consequent for each variable and obtaining the two end-points of interval set and average value of output.

Step 6: Calculate RMSE of this type-2 model. Choose the model with least RMSE.

2.4 Summary of Literature Review

The foregoing literature survey demonstrates the progress made in both type-1 TSK FLS and type-2 FL & TSK FLS to date. The results of investigated articles showed that:

- Study of type-1 TSK is much father than that of type-2;
- Type-1 TSK FLSs have much more application that of type-2;
- High order type-1 TSK FLSs are developed to overcome the problem of dimensionality;
- It seems that type-2 FL moves in progressive ways where type-1 FL is eventually replaced or supplemented by type-2 FL;
- Type-2 TSK FLS is a very new generation of TSK FLS.

It was very natural for fuzzy experts to learn about type-1 FL before type-2 FL, and develop type-1 TSK FLS as far as possible. Only by doing so was it really possible later to see the shortcomings of type-1 TSK FLS, and apply type-2 TSK FLS to situations where uncertainties abound. There are a lot of challenging problems waiting to overcome.
SECTION 3

RESEARCH METHODOLOGY

This section defines the problem of type-2 TSK FLS, sets up research objective and suggests relative methodology.

3.1 Problematic

Our subtractive clustering based type-2 TSK fuzzy system identification algorithm was already applied to function approximation [12] and identification of rigid-body dynamic of robotic manipulators [63]. Still, there are some problems to overcome:

- There is no theory that guarantees that a type-2 TSK FLS have the potential to outperform its type-1 counterpart;
- It is difficult to determine the search range for spreading percentage of cluster centers and consequence parameters to obtain an optimal model;
- Similar to type-1 rules of type-1 TSK system, first order type-2 TSK rules suffer the curse of dimensionality [40, 45] – the number of rules increases exponentially with the number of input variables and the number of MFs per variable.
3.2 Research Objectives

This research will focus on the problem: the curse of dimensionality. The research objectives include:

- propose the architecture, inference engine and design method of high order type-2 TSK FLS;
- analyze the fuzzy systems;
- apply to mechanical engineering.

3.3 Research Methodology

3.3.1 High order type-2 TSK Fuzzy Logic System

Higher order system modeling could identify a system without any parameter identification with smaller error for the same number of rules [40]. A larger reduction in the number of rules is possible with higher order system identification technique [1].

Comparing the $k$th rule in eq.(1.1) for high order type-1 FLS to that in eq.(1.2) of first order, observe that both system have similar structure. Their antecedents are the same, only variables are higher order in consequent equations. The difference between first order and higher order FLS is the computation of the consequent output for each rule. There are some work that have been done [1, 40-51] on high order type-1 TSK FLS. None is about type-2 TSK FLS.
Mendel in his book [7] proposes the architecture and a complete computation method for first order interval type-2 TSK FLS. It is possible generalize them to higher order type-2 TSK FLS.

Our type-2 TSK design method [16] works for first order TSK model using subtractive clustering method. Demirli has a paper on high order type-1 TSK FLS based on subtractive clustering [1], which is an extension of Chiu’s first order identification method [14, 15]. We want to propose a design method for high order type-2 TSK FLS based on Demirli’s high order type-1 TSK FLS using subtractive clustering.

The most difficult part in this research is the inference mechanism of high order type-2 TSK FLS. We should fully understand the structure identification and parameter identification of high order high order type-1 TSK FLS and the inference engine of first order type-2 TSK FLS, and create a complete new inference engine for high order type-2 TKS FLSs. In this kind of system, the antecedent or consequent MFs are type-2 fuzzy sets and its consequent part is a mathematical function that can be first-order or higher order polynomial function of input variables.

High order type-2 TSK systems have more of design parameters. It is anticipated that high order type-2 TSK FLS will have advantage of both TSK FLS: high order type-1 and first order type-2. It not only has the capability to overcome the problem of dimensionality, but also it can handle uncertainties within FLS, including measurement uncertainties.
3.3.2 Sensitivity analysis for type-2 TSK Fuzzy Logic System

Because there is no theory that guarantees that a type-2 TSK FLS have the potential to outperform its type-1 counterpart, in our subtractive clustering based type-2 TSK fuzzy system identification algorithm, always ask to compare the root-mean–square-error (RMSE) of type-1 and type-2 model. But it is still difficult to determine the search range for spreading percentage of cluster centers and consequence parameters to obtain an optimal model.

Good modeling practice requires that the modeler provides an evaluation of the confidence in the model, possibly assessing the uncertainties associated with the modeling process and with the outcome of the model itself. For type-2 FLS, no work has been done on this subject.

Sensitivity analysis is a simple technique to assess the effects of adverse changes on a model. It involves changing the value of one or more selected variables and calculating the resulting change in the output of the model. Sensitivity analyses are beneficial in determining the direction of future data collection activities.

The sensitivity analysis for its type-2 FLS aims to ascertain how RMSE, MFs and Model output of a type-2 TSK model depend upon spread percentage of cluster centres and consequent parameters.

3.3.3 Applications
As above mentioned type-2 TSK FLS has advantages: higher accuracy result and output interval which can be used to analyze the uncertainties associated with the experimental system. To demonstrate the performance of type-2 FLSs, Matlab programming will be done and type-2 FL will be applied in some field, in which uncertainties are present.

There are some suggested topics:

- Tool wear monitoring;
- Identification of rigid-body dynamic of robotic manipulators;
- Evaluation of inverse kinematics solution of redundant serial manipulators.
4.1 Expected Contributions

Our contributions in this research are as follow:

- propose the architecture, inference engine and design method of high order type-2 TSK FLS;
- create inference engine of high order type-2 TSK FLS;
- complete a design method for high order type-2 TSK FLS;
- sensitivity analyze the type-2 TSK fuzzy systems;
- application of type-2 TSK FLSs in mechanical engineering.

Results of this research work will contribute to uncertain rule-based FLS, which is a powerful design methodology and extremely important to effectively solve problems that are awash in uncertainties.

4.2 Research Schedule

Finally time chart for this research is presented as follows.
<table>
<thead>
<tr>
<th>Time Period</th>
<th>Task</th>
</tr>
</thead>
</table>
| Jan. 2007 – Jun. 2007 | • Literature review  
                        | • Preliminary test  
                        | • Conference: IASTED MS2007  
                        | NAFIPS’07 |
| Jun. 2007 – Aug. 2007 | • Propose the architecture, inference engine and design method of high order type-2 TSK FLS  
                        | • Create inference engine of high order type-2 TSK FLS |
| Sep. 2007 – Apr. 2008 | • Courses  
                        | • Complete a design method for high order type-2 TSK FLS |
| May 2008 – Dec. 2008  | • Matlab programming  
                        | • Sensitivity analyze  
                        | • Paper |
                        | • Paper |

Table 2 Time schedule
REFERENCES


[48] N. Yu, N. Y. Zhang, “Fuzzy sliding-mode control for higher-order SISO nonlinear systems”, Journal of Tsinghua University (Science and Technology), vol. 45, no. 10,
2005


[60] ----, “An Introduction to Type-2 Fuzzy Logic Systems”, 1999 IEEE International


