KINEMATIC ANALYSIS OF A 3-PSP SPATIAL MOTION PLATFORM

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OVERVIEW

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INTRODUCTION

Flight simulation training:

- not always require 6-DOF;
- most training require only 3-DOF;

Most important DOF:

- vertical motion;
- pitch rotation;
- roll rotation;

Least important DOF:

- yaw rotation;
- lateral motion;
- longitudinal motion;
3-PSP Platform

P = Prismatic
S = spherical
_P_ = actuated P

3-DOF
Key features:
- 3 parallel actuated P joints
- 3 passive P joints intersecting at \(120^\circ\)

Platform mobility:
1) vertical displacement (dof)
2) pitch rotation (dof)
3) roll rotation (dof)
4) dependent lateral displacement
5) dependent longitudinal displacement
6) no yaw rotation
Zero-Torsion manipulator
No yaw: $\sigma = 0$

$$
R(\theta, \phi) = \begin{bmatrix} r_{11} & r_{12} & r_{13} \\ r_{21} & r_{22} & r_{23} \\ r_{31} & r_{32} & r_{33} \end{bmatrix}
$$

$$
\begin{align*}
r_{11} &= \cos^2 \phi \cos \theta + \sin^2 \phi \\
r_{12} &= \sin \phi \cos \phi (\cos \theta - 1) \\
r_{13} &= \cos \phi \sin \theta \\
r_{21} &= \sin \phi \cos \phi (\cos \theta - 1) \\
r_{22} &= \sin^2 \phi \cos \theta + \cos^2 \phi \\
r_{23} &= \sin \phi \sin \theta \\
r_{31} &= -\sin \theta \cos \phi \\
r_{32} &= -\sin \theta \sin \phi \\
r_{33} &= \cos \theta.
\end{align*}
$$
Le génie en première classe

KINEMATIC MODEL : Position analysis

\[ \alpha_i \equiv 2(i - 1)\pi/3, \quad i = 1, 2, 3 \]

\[ b'_i = [r \cos \alpha_i \quad r \sin \alpha_i \quad 0]^T \]

\[ p' = [x' \quad y' \quad z']^T \]

\[ p = Rp' = [x \quad y \quad z]^T \]

Loop 1: \[ m'_i = a'_i + q'_i = a_i n'_i + q_i k'_i \]

\[ n_i = [\cos \alpha_i \quad \sin \alpha_i \quad 0]^T \]

\[ n'_i = R^T n_i. \]

Loop 2: \[ m'_i = p' + b'_i \]

Closer equation: \[ \det \begin{bmatrix} m'_i & n'_i & k'_i \end{bmatrix} = 0 \]

\[ k'_i = [0 \quad 0 \quad 1]^T \]
KINEMATIC MODEL : Position analysis

\[
x = \frac{r \cos 2\phi + 2z \sin \theta \cos \phi}{2 \cos \theta} - \frac{1}{2} r \cos 2\phi
\]

\[
y = r \sin \phi \cos \phi - \frac{\sin \phi (r \cos \phi - z \sin \theta)}{\cos \theta}
\]

(c)
Inverse kinematic solution

\[
q_1 = \frac{z - r \sin \theta \cos \phi}{\cos \theta}, \\
q_2 = \frac{2z + r \sin \theta \cos \phi - \sqrt{3}r \sin \theta \sin \phi}{2 \cos \theta}, \\
q_3 = \frac{2z + r \sin \theta \cos \phi + \sqrt{3}r \sin \theta \sin \phi}{2 \cos \theta}
\]  \hspace{1cm} (12)

The time derivative of eq. (12) gives

\[
A \dot{\pi} = B \dot{q}
\]

\[
\dot{\pi} \equiv [\dot{z} \dot{\theta} \dot{\phi}]^T, \quad \dot{q} \equiv [\dot{q}_1 \dot{q}_2 \dot{q}_3]^T.
\]  \hspace{1cm} (15)

\[
A(z, \theta, \phi) = \begin{bmatrix}
a_{11} & a_{12} & a_{13} \\
a_{21} & a_{22} & a_{23} \\
a_{31} & a_{32} & a_{33}
\end{bmatrix}, \quad B = 1_{3 \times 3},
\]  \hspace{1cm} (16)

\[
a_{11} = \frac{1}{\cos \theta}, \\
a_{12} = \frac{z \sin \theta - r \cos \phi}{\cos^2 \theta}, \\
a_{13} = \frac{r \sin \theta \sin \phi}{\cos^2 \theta}, \\
a_{21} = \frac{1}{\cos \theta}, \\
a_{22} = \frac{r (\cos \phi - \sqrt{3} \sin \phi) + 2z \sin \theta}{2 \cos^2 \theta}, \\
a_{23} = \frac{-r \sin \theta (\sin \phi + \sqrt{3} \cos \theta)}{2 \cos \theta}, \\
a_{31} = \frac{1}{\cos \theta}, \\
a_{32} = \frac{r (\cos \phi + \sqrt{3} \sin \phi) + 2z \sin \theta}{2 \cos^2 \theta}, \\
a_{33} = \frac{r \sin \theta (-\sin \phi + \sqrt{3} \cos \theta)}{2 \cos \theta}.
\]
1) First kind of singularity: occur at the boundary of the workspace or an internal boundary limiting subregions having different number of branches.

\[ B = 1_{3 \times 3}, \quad \text{det}(B) = 3 \neq 0. \]

2) Second kind of singularity: occur when the platform is locally movable even when its actuated joints are locked. At these locations, the platform gains one or more degrees of freedom.

\[ \text{det}(A) = 0 \]

3) Third kind of singularity: occur when both serial and parallel Jacobian matrices become simultaneously singular, i.e., when the position relationship from which we derive the velocity equation degenerates.

\[ A = \frac{1}{\cos \theta} A^* \]

\( A \) degenerates when \( \theta = \pi/2 \)
FLIGHT SIMULATION PLATFORM

Flight simulation software:

a) x-plane.org
b) Flightgear.org
CONCLUSIONS

Aircraft Simulation Laboratory:

- CAE-AEE software
- realistic real-time simulation
- bi-motors commercial jet

Needs connection to:

- a visual ...
- a motion platform
- a yoke
- a rudder pedals