ABSTRACT

This paper presents the kinematic analysis of a 3-PSP motion seat to be used as a flight simulator. To ensure the pilot’s security, all passive joints are kept on the base plate rather than under the moving platform, and thus, use an upside-down configuration relative to previously published results in the literature. Although the resulting kinematic is more complex, it can still be solved in an analytical form, even when a non-symmetric base and moving platforms are used.

NOMENCLATURE

A Frame attached to body A, i.e., the base.
B Frame attached to body B, i.e., the platform.
Ai Attachment points of leg i on body A.
Bi Attachment points of leg i on body B.
p Position vector of the origin of B relative to A.
a Position vector of Ai relative to A.
b Position vector of Bi relative to B.
m Position vector of Bi relative to A.
qi Position vector of Bi relative to Ai.
k Unit vector along qi, the actuated prismatic joint.
n Unit vector along ai, the passive prismatic joint.
qi Actuated prismatic displacement or norm of qi.
ai Passive prismatic displacement or norm of ai.

INTRODUCTION

For flight simulation applications, parallel manipulators are first-choice mechanisms to provide the 6-degrees-of-freedom (dof) mobility of the motion platform. In general, these manipulators are particularly worthy of note because they have a high carrying capacity, as well as a lower workspace volume, more singularity problems and an increased complexity when solving the direct kinematic problem compared to serial manipulators of equivalent size [1]. However, since the full 6-dof is not required for every type of flight simulation training, the use of lower mobility mechanisms may often fulfill the same training objectives at a much lower cost. In particular, 3-dof motion platforms providing a vertical displacement and pitch and roll rotations, without either the lateral and longitudinal displacement or the yaw ro-

Figure 1. MOBILITIES OF AN AIRCRAFT.
tation (see Fig. 1), are the most important mobilities [2] required from the point of view of flight simulation. Like 3-PSP platforms, these mechanisms have many advantages in terms of simplicity of construction/control and reduced cost, although they also present some problems due to the coupled orientation and position of the platform. Most, if not all, of the research work published in the literature on these platforms has been done on the 3-PSP variant [4–8], i.e., where the actuated prismatic joint is located on the base, unlike our variant 3-PSP, where the actuated prismatic joint is located on the platform.

In such a design, all passive joints (possibly harmful) are kept on the base and away from the pilot’s seat to ensure the pilot’s security, make the flight simulator easy to build, and accordingly, obtain a configuration that is upside-down relative to the studies published in the literature.

MECHANICAL SYSTEM

As Fig. 2 shows, the flight simulator’s 3-PSP motion platform has a fixed base plate lying on the floor and a moving platform, shown here as the tubing structure holding the pilot seat. Three motorized ball screws are rigidly attached under the structure, pointing toward the base plate. Given that the three ball screws are parallel to each other, they provide three independent displacements. In fact, the moving seat acts as a chair with three parallel legs of variable lengths. Obviously, an equal displacement of the three leg lengths provides a vertical translation of the seat at constant orientation, while a different displacement of the three legs provides a vertical displacement combined with side and forward displacements as well as a change of orientation. For this type of chair, the leg tips need to slip on the floor in order to allow a different displacement of the leg lengths.

Therefore, the end-point of each ball screw of the motion platform is attached to the base plate through a sphere sliding into a horizontal cylindrical hole with an open top, as shown in Fig. 3. The actuated ball screwed together with its spherical end-point is the SP joints. The square part screwed to the base plate and having a horizontal hole with an open top is the passive prismatic P joint. The center of the sphere can apparently freely slide along the hole axis and is therefore equivalent to the required passive slipping of the leg tip on the floor.

To obtain the full 3-dof (degree of freedom) of the motion platform, the orientation of the three hole axes cannot be arbitrary. In fact, we chose to locate these three axes on the base plate within a plan parallel to the floor, intersecting at one point and equally spaced at 120°. We also chose to locate the three ball-screw axes on the motion platform at the vertex of an equilateral triangle and normal to that triangle.

PLATFORM MOBILITY

Our 3-PSP mechanism must provide only 3-dof of mobility for the platform, irrespective of whether we obtain a combined motion. When the three actuated ball screws produce different displacements, the platform moves vertically together tilting the latter. At the same time, we can observe slight side and forward
displacements which indicate relationships between the tilting and these so-called parasitic displacements. Moreover, the particular choice of orientation of the three passive prismatic joint axes allows for only 2-dof of orientation, i.e., a rotation around an axis always lying in the horizontal plan. In summary, the platform has 1-dof of vertical displacement together with 2-dof of rotation around a horizontal axis, while the side and forward displacements are dependent onto the orientation.

The till-and-torsion orientation representation [3] is particularly useful in this case of orientation mobility. As shown in Fig. 4(a), the angle \( \theta \), called tilt, is a rotation around axis \( a \) located in the original xy-plane. The orientation of this axis is given by the angle \( \phi \), called azimuth, which is the angle between the projection of \( z^* \) onto the original xy-plane and the original x-axis. As Fig. 4(b) illustrates, the third angle \( \sigma \), called torsion, is a rotation about \( z^* \).

It has been pointed out [3] that 3-PSP mechanisms always have a zero torsion angle, i.e., \( \sigma = 0 \), and are hence called zero-torsion mechanisms. In this situation, the orientation of the platform can be expressed by the following rotation matrix, i.e.,

\[
R(\theta, \phi) = \begin{bmatrix}
  r_{11} & r_{12} & r_{13} \\
  r_{21} & r_{22} & r_{23} \\
  r_{31} & r_{32} & r_{33}
\end{bmatrix},
\]

(1)

with

\[
\begin{align*}
  r_{11} &= \cos^2 \phi \cos \theta + \sin^2 \phi \\
  r_{12} &= \sin \phi \cos \theta (\cos \phi - 1) \\
  r_{13} &= \cos \phi \sin \theta \\
  r_{21} &= \sin \phi \cos \theta (\cos \phi - 1) \\
  r_{22} &= \sin^2 \phi \cos \theta + \cos^2 \phi \\
  r_{23} &= \sin \phi \sin \theta \\
  r_{31} &= -\sin \theta \cos \phi \\
  r_{32} &= -\sin \theta \sin \phi \\
  r_{33} &= \cos \theta.
\end{align*}
\]

(2)

where the torsion angle \( \sigma \) has been assigned to zero, i.e. \( \sigma = 0 \). In order to avoid the singularity at \( \theta = \pi \), we set the range of azimuth and tilt angles to \( \phi \) \in \([-\pi, \pi]\) and \( \theta \in \left[0, \pi\right] \), respectively.

**KINEMATIC MODEL**

As Fig. 5 shows, the 3-PSP motion platform consists of two main bodies: a fixed base \( A \), and a mobile platform \( B \), under which three actuated ball screws are rigidly attached to points \( B_i \) with \( i = 1, 2, 3 \). The axis of each actuator is normal to the triangle formed by points \( B_1, B_2, B_3 \). Each leg is attached to the base plate via a passive spherical joint located at point \( A_i \). Our prototype uses an equilateral triangle \( B_1, B_2, B_3 \) on the platform and three coplanar intersecting axes at 120° for the passive prismatic joints on the base. It is obviously possible to use none-equilateral triangles for the base and the platform.

Without loss of generality, let us attach frame \( A \) to body \( A \), its origin located at the intersection point of the three passive prismatic joints, its \( z \)-axis normal to the plan formed by these three prismatic joints and its \( x \)-axis oriented toward point \( A_1 \). Similarly, let us attach frame \( B \) to body \( B \), its origin located at the center of the three actuated prismatic joints, its \( z \)-axis parallel to these joints axes and its \( x \)-axis pointing toward point \( B_1 \). The position along the \( z \)-axis is arbitrarily chosen. Since the actuated prismatic joints are acting along the \( z \)-axis of frame \( B \), let us develops the kinematic relationships into frame \( B \) before trans-
erring them into frame $A$. In addition, let us denote with a prime every vector expressed in the moving frame $B$. Alternatively, any vector without a prime is by default expressed in frame $A$.

**Position Analysis**

The position vector of point $B_i$ is expressed in $B$ as

$$b_i' = [r \cos \alpha_i \ r \sin \alpha_i \ 0]^T,$$  

with $\alpha_i$ being defined as

$$\alpha_i \equiv 2(i-1)\pi/3, \ i = 1, 2, 3.$$  

In the base frame $A$, the unit vectors along the three passive prismatic joints can be written as

$$n_i = [\cos \alpha_i \ \sin \alpha_i \ 0]^T,$$  

or alternatively, in the moving frame $B$, as

$$n_i' = R^T n_i.$$  

The position vector of the origin of $B$ relative to the origin of $A$ expressed in the moving frame $B$ is given as

$$p' = [x' \ y' \ z']^T,$$  

and alternatively, in the base frame $A$ as

$$p = Rp' = [x \ y \ z]^T.$$  

The position vector of point $B_i$ relative to the origin of $A$, but expressed in $B$, is given as

$$m_i' = p' + b_i'.$$  

Alternatively, the same position vector can also be obtained as

$$m_i' = a_i' + q_i' = a_i n_i' + q_i k_i'.$$  

Since the closure equation requires vectors $m_i'$, $n_i'$ and $k_i'$ of each individual leg $i$ to be coplanar, they must satisfy the following equation

$$\det [m_i' \ n_i' \ k_i'] = 0,$$  

where $k_i' = [0 \ 0 \ 1]^T$. Using eq.(11), we can algebraically obtain the solution of the *inverse kinematic problem*, i.e., the actuated joint position $q_i$ as

$$q_1 = \frac{z - r \sin \theta \cos \phi}{\cos \theta},$$  
$$q_2 = \frac{2z + r \sin \theta \cos \phi - \sqrt{3}r \sin \theta \sin \phi}{2 \cos \theta},$$  
$$q_3 = \frac{2z + r \sin \theta \cos \phi + \sqrt{3}r \sin \theta \sin \phi}{2 \cos \theta}$$  

from a given position and orientation of the platform, i.e., $z$, $\theta$, $\phi$. It is worth noting that $z$ is an independent coordinate that can be chosen freely, while $\theta$ and $\phi$ are the specified orientation of the platform. An important feature of this mechanism is the parasitic displacement in $x$ and $y$, which is related to the platform’s position and orientation, i.e., $z$, $\theta$ and $\phi$ as

$$x = \frac{r \cos 2\phi + 2z \sin \theta \cos \phi}{2 \cos \theta} - \frac{1}{2}r \cos 2\phi,$$
$$y = r \sin \phi \cos \phi - \frac{\sin \phi (r \cos \phi - z \sin \theta)}{\cos \theta}$$  

**Velocity Analysis**

On differentiating eq.(12), it is easy to discover the mechanism’s general velocity realtionship, i.e.,

$$A\dot{\pi} = B\dot{q}$$  

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where $\dot{\pi}$, namely the twist of the platform, and $\dot{q}$, namely the actuated joint velocities, are defined as

$$\dot{\pi} = [\dot{\gamma} \; \dot{\theta} \; \dot{\phi}]^T, \quad \dot{q} = [\dot{q}_1 \; \dot{q}_2 \; \dot{q}_3]^T. \quad (15)$$

Matrices $A$ and $B$ are the so-called parallel and serial Jacobian matrices given as

$$A(z, \theta, \phi) = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}, \quad B = I_{3\times3}, \quad (16)$$

with

$$a_{11} = \frac{1}{\cos \theta},$$

$$a_{12} = \frac{z \sin \theta - r \cos \phi}{\cos^2 \theta},$$

$$a_{13} = \frac{r \sin \theta \sin \phi}{\cos^2 \theta},$$

$$a_{21} = \frac{1}{\cos \theta},$$

$$a_{22} = \frac{r (\cos \phi - \sqrt{3} \sin \phi) + 2z \sin \theta}{2 \cos \theta},$$

$$a_{23} = \frac{-r \sin \theta (\sin \phi + \sqrt{3} \cos \theta)}{2 \cos \theta},$$

$$a_{31} = \frac{1}{\cos \theta},$$

$$a_{32} = \frac{r (\cos \phi + \sqrt{3} \sin \phi) + 2z \sin \theta}{2 \cos \theta},$$

$$a_{33} = \frac{r \sin \theta (-\sin \phi + \sqrt{3} \cos \theta)}{2 \cos \theta}. \quad (17)$$

### Singularity Analysis

Singularities occur when the mechanical system reaches configurations where matrix $A$ or $B$ becomes singular [9]. A distinction must be made between three kinds of singularities because they have very different physical interpretations.

1) The first kind of singularity occurs when the serial Jacobian matrix $B$ satisfies the following condition, i.e.,

$$\det(B) = 0. \quad (18)$$

This type of singularity occurs either at the boundary of its workspace or at an internal boundary limiting different sub-regions of the workspace having different number of branches [9]. In our case, $B = I_{3\times3}$, and hence, $\det(B) = 3 \neq 0$.

2) The second kind of singularity occurs when the parallel Jacobian matrix $A$ satisfies the following condition, i.e.,

$$\det(A) = 0. \quad (19)$$

This type of singularity occurs when the end-effector (here the platform) is locally movable even when its actuated joint is locked. At these workspace locations, the end-effector gains one or more degrees of freedom, and thus cannot resist one or more forces/moments.

3) The third kind of singularity occurs when the both serial and parallel Jacobian matrices become simultaneously singular. This occurs when the position relationship, i.e., eqs.(12) from which we derive the velocity equation (14), degenerates. This corresponds to configurations in which the end-effector can undergo finite motions with its actuators locked or in which a finite motion of the actuators produces no motion of the end-effector.

In our case, it is possible to factorize $1/\cos \theta$ from eqs.(17) of $A$ such that

$$A = \frac{1}{\cos \theta} A^*, \quad (20)$$

and hence, $A$ degenerates when $\theta = \pi/2$. At this tilting angle, the legs become of infinite length, and do not produce any motion of the platform. Obviously, the mechanism is not able to reach such a high value of $\theta$. In fact, our prototype is able to reach a maximum tilt angle of $\pi/6$. The actual implementation of the flight simulator is shown in Fig. 6.

### DIRECT KINEMATICS

The direct kinematic problem aimed at computing the platform position and orientation, i.e $z, \theta, \phi$, for a given set of actuated joint positions, i.e., $q_1, q_2, q_3$. Because of the unknown position of the passive joints, it is easier to formulate the problem in frame $B$ and transform its final results in the base frame $A$.

The position vector of point $A_i$ relative to the origin of $B$, and expressed in $B$, is given as

$$c_i' = b_i' + q_i'. \quad (21)$$

Vector $n'$ normal to the plan, namely $\gamma$, passing through points $\{A_i\}^3_i$ can be computed as

$$n' = (c_1' - c_1') \times (c_2' - c_1'). \quad (22)$$
On substituting eqs.(3) and (21) into (22), we obtain

\[ n' = \begin{bmatrix} n'_x \\ n'_y \\ n'_z \end{bmatrix} = \begin{bmatrix} \frac{\sqrt{3}}{2} r(q_2 - 2q_1 + q_3) \\ \frac{3}{2} r(q_2 - q_3) \sqrt{3} \frac{r}{r^2} \end{bmatrix}. \] (23)

The equation of the \( \gamma \) plan is thus given as

\[ n'_x u + n'_y v + n'_z w - \frac{\sqrt{3}}{2} r^2 (q_1 + q_2 + q_3) = 0 \] (24)

where \( (u, v, w) \) are the coordinates of the points lying into the plan. The output \( z \) coordinate can be computed as the distance, along the \( z \)-axis of \( \mathcal{B} \), between the plan \( \gamma \) and the origin of \( \mathcal{B} \), i.e.,

\[ z = \frac{r(q_1 + q_2 + q_3)}{\sqrt{3(q_2 - q_3)^2 + (q_2 - 2q_1 + q_3)^2 + 9r^2}}. \] (25)

The output tilt angle \( \theta \) is now computed as the angle between vectors \( n' \) and \( k' = [0 0 1]^T \), i.e.,

\[ \theta = \arccos \left( \frac{3r}{\sqrt{3(q_2 - q_3)^2 + (q_2 - 2q_1 + q_3)^2 + 9r^2}} \right). \] (26)

Finally, the output azimuth angle \( \phi \) is computed from eq.(12), i.e.,

\[ \phi = \arctan2(\sin \phi, \cos \phi), \] (27)

with

\[ \sin \phi = \frac{\frac{1}{2} q_3 \cos \theta + 2r + 2q_2 \cos \theta}{\sqrt{3} \sin \theta}, \]
\[ \cos \phi = \frac{z - q_1 \cos \theta}{r \sin \theta}. \] (28)

**WORKSPACE**

The workspace of the 3-dof motion platforms is determined by varying the following three variables \( z, \phi \) and \( \theta \), and computing the corresponding joint positions \( \{ q_i \}^3 \) with eqs.(12). The first loop varies \( z \) from 0 to \( \{ q_i \}_{\max} \). The second loop varies \( \phi \) from \(-\pi\) to \(+\pi\), and the third loop varies \( \theta \) from 0 to the angles for which one of the \( q_i \) reaches its minimum or maximum. For \( r = 100 \text{ mm} \) and \( \{ q_i \}_{\max} = 100 \text{ mm} \), Fig. 7 shows different views of the Cartesian displacement of the origin of \( \mathcal{B} \) in \( \mathcal{A} \). Apparently, the amplitudes of the parasitic displacement in \( x \) and \( y \) are must smaller than the vertical displacement in \( z \), making their contributing to the simulation almost irrelevant. Moreover, these parasitic displacement always appear together with a roll and/or pitch rotation as expressed by eq.(13).

**CONCLUSIONS**

The 3-dof motion platforms are particularly well-suited for the flight simulation applications. The 3-PSP variant with the three actuated prismatic joints rigidly attached under the platform is secure for the pilot and easy to construct. It remains possible to solve its kinematic in an analytical form, even when a non-symmetric base and moving platforms are used.

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Figure 7. THE WORKSPACE OF THE 3-PSP FOR \( r = 200mm \) AND \( \{q_i\}_{\text{max}} = 100mm \).