SENSITIVITY ANALYSIS FOR TYPE-1 AND TYPE-2 TSK FUZZY MODELS

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ABSTRACT
In this paper, subtractive clustering method is combined with least squares estimation algorithms to pre-identify a type-1 Takagi-Sugeno-Kang (TSK) fuzzy model from input/output data. Then the type-2 fuzzy theory is used to expand the type-1 model to a type-2 model. A sensitivity analysis is used to ascertain how a type-1 TSK model output depends upon the pre-initialized parameters and determine how a type-2 TSK model output depends upon spread percentages of cluster centers and consequent parameters. By using sensitivity analysis, we can check the quality of TSK models, and characterize the uncertainty associated with the TSK fuzzy models.

KEY WORDS
Fuzzy logic system, subtractive clustering, modeling, sensitivity analysis

1. Introduction
Fuzzy modeling is the most important issue in fuzzy logic or more widely in fuzzy theory. The fuzzy logic (FL) has been originally proposed by Zadeh in his famous paper “Fuzzy Sets” in 1965 [1]. Later on, his proposal of linguistic approach [2, 3] is expanded into fuzzy systems modeling as qualitative modeling. Qualitative modeling has the capability to model complex system behavior in such a qualitative way that the model is more effective and versatile in capturing the behavior of ill-defined systems with fuzziness or fully defined system with realistic approximation.

Takagi-Sugeno-Kang (TSK) qualitative modeling based on fuzzy logic [4, 5], as known as TSK modeling, was proposed in an effort to develop a systematic approach to generating fuzzy rules from a given input-output data set. This model consists of rules with fuzzy antecedents and mathematical function in the consequent part. The antecedents divide the input space into a set of fuzzy regions, while consequents describe behavior of system in those regions.

TSK models are widely used for model-based control and model-based fault diagnosis. This is due to the model’s properties of, on one hand being a general nonlinear approximator that can approximate every continuous mapping, and on the other hand being a piecewise linear model that is relatively easy to interpret [6] and whose linear submodels can be exploited for control and fault detection [7, 8].

In general, the system identification has to be automatic in order to make it more versatile and suitable for future learning. There is a need to develop a method to evolve the system model using input and output data of the system. The aim of clustering methods is to identify natural grouping of data from a large data set, such that a concise representation of system’s behaviour is produced. In literature different modeling techniques can be found [9]. Chiu’s subtractive clustering [10] is an extension of the grid-based mountain clustering methods [11]. Chiu’s method operates by finding the optimal data point to define a cluster centre based on the density of surrounding data points. This method reduces the computational complexities and gives better distribution of cluster centers in comparison with other clustering algorithms.

Based on the extension principle [3], Mendel and his co-authors extended previous studies and established a complete type-2 fuzzy logic theory with the handling of uncertainties [12]. Type-2 TSK FLS was presented in 1999 [13]. Type-2 TSK FLSs have the potential to be used in control and other areas where a type-1 TSK model may be unable to perform well [12] because of its large numbers of design parameters.

According type-2 fuzzy theory [12], a type-2 TSK fuzzy model identification algorithm based on subtractive clustering [14] is introduced. The type-2 model is
obtained by considering the type-1 membership functions (MFs) as principle MFs and assigning uncertainties to cluster centers, consequence parameters and standard deviations of Gaussian MF. Minimum error models are obtained through enumerative search of optimum values for spreading percentage of cluster centers and consequence parameters.

Good modeling practice requires that the modeler provides an evaluation of the confidence in the model, possibly assessing the uncertainties associated with the modeling process and with the outcome of the model itself. Sensitivity analysis is a simple technique to assess the effects of adverse changes on a model. It involves changing the value of one or more selected variables and calculating the resulting change in the output of the model.

In this paper, section 2 recalls initially subtractive clustering based type-1 TSK model and section 3 introduces type-2 TSK model. Section 4 and 5 are sensitivity analysis of type-1 and type-2 TSK model through a function approximation example. The aim of sensitivity analysis of type-1 model is to estimate the change in least squares error (LSE) of subtractive clustering based TSK modelling with respect to changes in pre-initialized parameters. The sensitivity analysis of type-2 model aims to ascertain how the root-means-square-error (RMSE) of type-2 TSK model depends upon spread percentage of cluster centers and consequent parameters. Changes in parameters are assessed once at a time to identify the key parameters. Finally, the concluding remarks are given in Section 6.

Although higher order type-1 TSK models have been described in the literature, this paper focuses exclusively on first-order type-1 TSK models multi-input single-output (MISO) system, because they are the most widely used and are easily extended to type-2 TSK models.

2. Type-1 TSK model

In general, TSK models can be classified based on the type of antecedent MFs and consequent parameters [3]. As shown in Table 1, there are four structures for TSK models because of the possible type-1 or type-2 natures of the antecedent memberships and consequent parameters for their rules: one for type-1 TSK model and three for type-2 model.

Table 1. Classification of TSK models

<table>
<thead>
<tr>
<th>TSK model type</th>
<th>Antecedent MF type</th>
<th>Consequent parameter type</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>type-1 fuzzy sets</td>
<td>type-2 fuzzy sets</td>
</tr>
<tr>
<td>Consequent parameter type</td>
<td>type-1 fuzzy sets</td>
<td>Type-2 Model III</td>
</tr>
<tr>
<td>crisp numbers</td>
<td>Type-1</td>
<td>Type-2 Model II</td>
</tr>
</tbody>
</table>

A generalized type-1 TSK model can be described by fuzzy IF-THEN rules which represent input-output relations of a system; its kth rule can be expressed as:

\[
\text{IF } x_1 \text{ is } Q^k_1 \text{ and } x_2 \text{ is } Q^k_2 \text{ and } \ldots \text{ and } x_n \text{ is } Q^k_n, \text{ THEN } Z \text{ is } w^k = p^k_0 + p^k_1 x_1 + p^k_2 x_2 + \ldots + p^k_n x_n
\]

where \( x_1, x_2, \ldots, x_n \) and \( Z \) are linguistic variables; \( Q^k_1, Q^k_2, \ldots, Q^k_n \) are the fuzzy sets on universe of discourses \( U, V, \ldots, W \), and \( p^k_0, p^k_1, p^k_2, \ldots, p^k_n \) are regression parameters.

Subtractive clustering based type-1 TSK modeling [10] involves generation of clusters using input/output data set for pre-initialized clustering parameters in Table 2, and estimation of regression coefficients by least-square estimation.

Table 2. Parameters in subtractive clustering

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
</thead>
<tbody>
<tr>
<td>( r )</td>
<td>Hypersphere cluster radius in data space</td>
</tr>
<tr>
<td>( \eta )</td>
<td>Squad factor ( \eta = \frac{r}{r_{center}} )</td>
</tr>
<tr>
<td>( \sigma )</td>
<td>Standard deviation of Gaussian MF</td>
</tr>
</tbody>
</table>

Since, Gaussian basis functions (GBFs) have the best approximation property [15], these functions are chosen as the MFs. A type-1 Gaussian MF can express by using formula for the vth variable:

\[
Q^k_v = \exp \left[ -\frac{1}{2} \left( \frac{x_v - x_v^*}{\sigma} \right)^2 \right]
\]

where \( x_v^* \) is mean of the vth input feature in the kth rule for \( v \in [0,n] \). The standard deviation of Gaussian MF \( \sigma \) is given as

\[
\sigma = \sqrt{\frac{1}{2\alpha} \cdot r_v}
\]

with \( \alpha = \frac{4}{r_v^2} \). \( r_v \) is cluster radius defined in Table 2.
Subtractive clustering for type-1 TSK model is evaluated by least square error:

\[ LSE = \sum_{j=1}^{n} (W_{j\mu} - W_{j\nu})^2 \quad (4) \]

where the initial system has a group of data with \( n \) vector, \( W_{j\mu} \) and \( W_{j\nu} \) are the system output and model output for \( j \)th vector, \( j \in [1,n] \).

Based only on measured data and without prior knowledge, there is no systematic way to obtain a TSK model with a simple structure and sufficient accuracy. By using subtractive clustering, it is easy to obtain an efficient Type-1 TSK model. But there are still some uncertainties on the algorithm. For TSK models, the uncertainties are about the meanings of words by using precise membership functions, the consequent that is used in a rule are extracted directly from data which is leading to a histogram of possibilities. Type-2 TSK model can handle uncertainties because it can model them and minimize their effects [12].

3. Type-2 TSK model


A generalized \( k \)th rule in the first-order type-2 TSK fuzzy system can be expressed as eq. (5) instead of eq.(1) in type-1 system.

\[ \text{IF } x_1 \text{ is } Q_1^k \text{ and } x_2 \text{ is } Q_2^k \text{ and } \ldots \text{ and } x_n \text{ is } Q_n^k, \]

\[ \text{THEN } z \text{ is } w = p_0 + p_1 x_1 + p_2 x_2 + \ldots + p_n x_n \quad (5) \]

where \( p_0, p_1, \ldots, p_n \) are consequent parameters, \( w \) output from the \( k \)th IF-THEN rule in a total of \( M \) rules FLS, \( Q_1^k \), \( Q_2^k \), \ldots, and \( Q_n^k \) are fuzzy sets on universe of discourses.

In the type-2 TSK fuzzy model identification algorithm [14], in order to obtain a type-2 model directly from a type-1 model, a width \( a_j^k \) of cluster center \( x_j^{k\mu} \) is extended to both two directions of cluster center \( x_j^{k\mu} \), as shown in Fig. 1. By doing so, cluster centers are expanded from a certain point to a fuzzy number:

\[ x_j^k = [x_j^{k\mu}(1-a_j^k), x_j^{k\mu}(1+a_j^k)] \quad (6) \]

where \( a_j^k \) is the spread percentage of cluster center \( x_j^{k\mu} \) as shown in Fig.1. The cluster center \( x_j^{k\mu} \) becomes a constant width interval valued fuzzy set \( x_j^k \).

Consequent parameters are obtained by expanding consequent parameters from its type-1 counterpart to fuzzy numbers by eq.(8) where \( b_j^k \) is the spread percentage of fuzzy numbers \( p_j^k \).

\[ p_j^k = p_j^k(1 \pm b_j^k) \quad (8) \]

Because that the staring point for the least-squares method to design a type-1 TSK FLS is a type-1 fuzzy basis function expansion [15], the performance of a type-2 TSK FLS is evaluated using the following RMSE:

\[ \text{RMSE} = \sqrt{\frac{1}{n} \sum_{j=1}^{n} (W_{j\mu} - W_{j\nu})^2} \quad (9) \]

4. Sensitivity analysis of type-1 TSK modelling

Subtractive clustering based type-1 TSK modelling typically contains pre-initialized parameters, and the numerical results can be highly sensitive to small changes in the parameter values. In this paper a function approximation example is used for sensitivity analysis of both type models.

The following system
$y = (x-2.5)^2 + x + 1 \quad \text{where} \quad x \in [0,4]$ \hspace{1cm} (10)

is used for sensitivity analysis for type-1 TSK model and its type-2 counterpart.

To estimate the change in LSE with respect to changes in pre-initialized parameters: cluster radius $r_a$, reject radius $\varepsilon$, accept radius $\overline{\varepsilon}$ and squash factor $\eta$. Figures 2 to 5 depict the influence of $r_a$, $\varepsilon$, $\overline{\varepsilon}$ and $\eta$ to LSE. The value of $r_a$, $\varepsilon$, $\overline{\varepsilon}$ and $\eta$ are chosen from $[0.15, 1]$, $[0, 0.9]$, $[0, 1]$ and $[0.05, 2]$, the step sizes are selected as $0.002, 0.1, 0.1$ and 0.05.

It is observed that the LSE of type-1 model is very sensitive for $r_a$ and $\eta$. In his paper [16], Demirli described in detail the influence of the value of those four parameters in Table 1 on performance of a type-1 TSK model.

Using recommended values from Chiu’s subtractive clustering method [10], $r_a = 0.25$, $\varepsilon = 0.15$, $\overline{\varepsilon} = 0.5$, $\eta = 1.25$, cluster centres and parameters of consequences are obtained. By using GBFs, a six-rule type-1 TSK model is identified as Table 3.

5. Sensitivity analysis of type-2 TSK modelling

As mention in section 3, in order to extend a type-1 TSK FLS to its type-2 counterpart with emphasis on interval set, antecedent MFs have to be changed from type-1 fuzzy sets to type-2 fuzzy sets. Consequent parameters have also to be changed from a certain number to a fuzzy number.

Based on the type-1 TSK model rules in Table 2, a six-rule type-2 TSK model can be expended by assigning
uncertainty $a_j$, $b_j$, and $\sigma_j$ to cluster centers, standard deviation of Gaussian MF and consequence parameters. In order to estimate how RMSE depends upon spread percentage of cluster centres and consequent parameters, Figures 6 to 8 depict influence. The value of $a_j$ and $b_j$ are chosen from [0, 40] and that of $\sigma_j$ is chosen from [0.2, 0.6].

It is observed that $a_j$ and $\sigma_j$, for which RMSE of type-2 model is relatively sensitive, would require future characterization, as opposed to $b_j$ for which RMSE of the model is relatively insensitive. The TSK model is very sensitive to uncertainty of cluster centre $a_j$ and $\sigma_j$.

Because the value of $a_j$ and $b_j$ decide the size of bounded regions of the union of all antecedent primary memberships – the footprint of uncertainty (FOU) which is the area between upper MF and lower MF of type-2 MF in Fig. 9.

The type-2 TSK model provides more information, not only crisp output as that of type-1 TSK model, but also the interval set of the output. This interval set of the output has the information about the uncertainties that are associated with the crisp output. Model output is very sensitive for uncertainty of consequent parameters $b_j$.

Table 4 summarizes influence of $a_j$, $b_j$ and $\sigma_j$ on type-2 model’s RMSE, model output and Gaussian MFs.

<table>
<thead>
<tr>
<th>influence</th>
<th>$\sigma_j$</th>
<th>$b_j$</th>
<th>$a_j$</th>
</tr>
</thead>
<tbody>
<tr>
<td>RMSE</td>
<td>Strong</td>
<td>No</td>
<td>Strong</td>
</tr>
<tr>
<td>Model output</td>
<td>Little</td>
<td>Strong</td>
<td>No</td>
</tr>
<tr>
<td>Gaussian MFs</td>
<td>Strong</td>
<td>No</td>
<td>Little</td>
</tr>
</tbody>
</table>

Based on the type-1 TSK model of Table 3, the best six-type-2 rules is obtained in Table 5. In this case, the range of $a_j$, $b_j$ and $\sigma_j$ selected for enumerative search are [0,
Table 5. Type-2 TSK FLS

<table>
<thead>
<tr>
<th>Rule</th>
<th>If $x$, then $z = p_1 \times x + p_0$</th>
</tr>
</thead>
</table>
| 1    | $\text{If } x = \exp\left(-\frac{1}{2}(x - 2.5 \times (0 \pm 22.857\%))^2\right), \text{ then}
\begin{align*}
z & = 2.984 \times x(1 \pm 26.31\%) - 3.995 \times (1 \pm 10.52\%) 
\end{align*}$ |
| 2    | $\text{If } x = \exp\left(-\frac{1}{2}(x - 1.5 \times (0 \pm 10.67\%))^2\right), \text{ then}
\begin{align*}
z & = 2.984 \times x(1 \pm 26.31\%) - 3.995 \times (1 \pm 10.52\%) 
\end{align*}$ |
| 3    | $\text{If } x = \exp\left(-\frac{1}{2}(x - 3.5 \times (0 \pm 2.832\%))^2\right), \text{ then}
\begin{align*}
z & = 2.984 \times x(1 \pm 26.31\%) - 3.995 \times (1 \pm 10.52\%) 
\end{align*}$ |
| 4    | $\text{If } x = \exp\left(-\frac{1}{2}(x - 0.812 \times (0 \pm 5.097\%))^2\right), \text{ then}
\begin{align*}
z & = 2.984 \times x(1 \pm 26.31\%) - 3.995 \times (1 \pm 10.52\%) 
\end{align*}$ |
| 5    | $\text{If } x = \exp\left(-\frac{1}{2}(x - 3.125 \times (0 \pm 5.879\%))^2\right), \text{ then}
\begin{align*}
z & = 2.984 \times x(1 \pm 26.31\%) - 3.995 \times (1 \pm 10.52\%) 
\end{align*}$ |
| 6    | $\text{If } x = \exp\left(-\frac{1}{2}(x - 1.433 \times (0 \pm 35.96\%))^2\right), \text{ then}
\begin{align*}
z & = 2.984 \times x(1 \pm 26.31\%) - 3.995 \times (1 \pm 10.52\%) 
\end{align*}$ |

30], [0, 30] and [0.2, 0.4], and the step sizes are selected as 0.001, 0.001 and 0.0001 and its RMSE is 0.0815.

6. Conclusion

Sensitivity analyses are useful in determining the direction of future data collection activities. During the preliminary experiments, it has been found that each of these clustering parameters ($r_a, \epsilon, \sigma, \eta$) has specific influence on clustering performance, and hence, on the type-1 model. The spread percentages $a_j$ and $b_j$ and deviation $\sigma$ have great influence on different factors of performance of a type-2 TSK model. It is recommended an enumerative search for them to get the optimal model. Especially smaller step sizes need to selected for $r_a, \eta$, $a_j$ and $b_j$, because systems are more sensitive to them.

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References