Multi-Objective Trajectory Planning of Mobile Robots Using Augmented Lagrangian

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Outline

• Introduction

• Robot and Constraints Modelling

• Non linear Programming Formulation

• Augmented Lagrangian with Projection

• Implementation on a Wheeled Mobile Robot

• Conclusions and Discussions
Introduction

Motion Planning Problem

Path planning:
Geometric path, shortest path, obstacle avoidance, passing through imposed poses.

Trajectory planning:
Includes velocities, accelerations, dynamic forces, energies, task and workspace requirements.

Offline Trajectory Planning Framework

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The study of object motion regardless to forces causing this motion

Non-Holonomic Constraints:

Is a kinematic constraints that cannot be integrated to obtain a geometric constraint

\[ A(q) = 0 \]

\[ A(q) = \begin{bmatrix}
  \sin \varphi & -\cos \varphi & 0 & 0 & 0 \\
  \cos \varphi & \sin \varphi & b & -r & 0 \\
  \cos \varphi & \sin \varphi & -b & 0 & -r 
\end{bmatrix} \]

\( q = [x, y, \varphi, \theta_L, \theta_R]^T \) generalized coordinates

\( q = S(q)u \quad S(q) \quad \text{such that} \quad A(q)S(q)u = 0 \)

\( u = [\dot{\theta}_R, \dot{\theta}_L]^T = [u_1, u_2]^T \) pseudo-velocity
Study of relationship between displacements, displacement rates, and accelerations characterizing the robot motion and torques and forces causing this motion.

Dynamic Model

\[ D(q) \ddot{q} + V(q, \dot{q}) = B(q)\tau + A^T(q)\hat{\lambda} \]

- **\( D(q) \)**: Inertia matrix
- **\( V(q, \dot{q}) \)**: Position and velocity dependent forces,
- **\( B(q) \)**: Input matrix
- **\( \tau = [\tau_R, \tau_L]^T = [\tau_1, \tau_2]^T \)**: Input vector
- **\( \hat{\lambda} \)**: Associated Lagrange multiplier to the kinematic constraint
The non-holonomic kinematic equation
\[ \dot{q} = S(q)u \]

Dynamic equation
\[ u = -D^{-1}(q)\left[ V(q, u) + B(q)\tau \right] \]

Let \[ x = [q, u]^T \]

State-space representation
\[ \dot{x}(t) = F(t)x(t) + G(t)N(x(t)) + B(t)\tau(t) \]

Approximated Discrete-Time Dynamic Model

Developed using Euler discretization
\[ x_{k+1} = F_{dk}x_k + G_{dk}N(x_k) + B_{dk}\tau_k \]
Robot and Constraints Modelling

Associated Constraints

Robot Constraints

Discrete dynamic equation:

\[ x_{k+1} = f_{d_k}(x_k, \tau_k, h_k) \]

Limits on:

- Right wheel angle: \[ \theta_{RMin} < \theta_k < \theta_{RMax} \]
- Left wheel angle: \[ \theta_{LMin} < \theta_k < \theta_{LMax} \]
- Steering angle: \[ \varphi_{Min} < \varphi_k < \varphi_{Max} \]

Torque limits:

\[ C = \left\{ \tau_k \in \mathbb{R}^{2N}, \tau_{Min} < \tau_k < \tau_{Max}, k = 0, \ldots, N - 1 \right\} \]
Robot and Constraints Modelling

Associated Constraints

**Task Constraints**

- **Sampling periods**
  \[ H = \{ h_k \in \mathbb{R}^+, \ h_{\text{Min}} < h_k < h_{\text{Max}} \} \]

- **Intermediate pose limits**
  \[ p_{1\text{Min}} < p_{1k} < p_{1\text{Max}} \]

- **Imposed passage points**
  \[ \| p - p_1 \| - T_{\text{PassThlp}} = 0 \quad l = 1, \ldots, L \]
  \[ \text{with } L \text{ is the number of imposed points} \quad T_{\text{PassThlp}} \text{ passage tolerance} \]

**Environment Constraints**

- **Obstacles avoidance function**
  \[ \Theta(p_n, p_k) = \| p_n - p_k \| \quad n = 1, \ldots, N \text{ obstacles to avoid} \]
  \[ \text{This is expressed by an inequality constraint} \quad \Theta(p_k, p_n) \geq \eta^{ER} \]
  \[ \text{with } \eta^{ER} \text{ Avoidance tolerance} \]
Non linear Programming Formulation

Performance Index

**Performance Index:** Electric Power and Time Consumption

**Discrete Multi-objective Optimal Control Problem**

Find control inputs

\[ \mathbf{\tau}^k = (\tau^1_k, \tau^2_k)^T \quad \text{and} \quad h_k, \quad k = 0, \ldots, N - 1, \]

Solution to

\[ \begin{align*}
\text{Min} \quad & E_d = \sum_{k=0}^{N-1} \left[ l_1 K_{DC} (\tau^2_{1k} + \tau^2_{2k}) + l_2 \right] h_k \\
\text{subject to} \quad & (\mathbf{\tau}_1, \mathbf{\tau}_2) \in \mathbb{R}^{2N} \\
& h \in \mathbb{R}^{+N}
\end{align*} \]

with

\[ K_{DC} = \frac{R_e K^2}{K^2_e} \]

\( \tau^i \): torque applied to the \( i \)th motor.

\( K_{\phi} \): motor gear ratio, \( R_e \): motor resistance, \( K^2_e \): torque constant

\( l_1, l_2 \) : Electric Power and Time Level-Headedness Positive scalars
Non linear Programming Formulation

Augmented Lagrangian

\[ L_\mu(x, \tau, h, \lambda, \rho, \sigma) = \]

\[ \sum_{k=0}^{N-1} \left[ l_1 K_{DC} (\tau_{1k}^2 + \tau_{2k}^2) + l_2 \right] h_k + \]

\[ \sum_{k=0}^{N-1} \left[ \lambda_{k+1}^T (x_{k+1} - f_d (x_k, \tau_k, h_k)) \right] + \]

\[ \sum_{k=0}^{N-1} h_k \left[ \sum_{l=1}^{L-1} \sum_{i=1}^{2} \Psi_{\mu_S} (\sigma_i^k, s_i^l(x_k)) \right] + \]

\[ \sum_{k=0}^{N-1} h_k \left[ \sum_{j=1}^{J} \Phi_{\mu_g} (\rho_j, g_j (x_k, \tau_k, h_k)) \right] \]

**Penalty functions**

\[ \Psi_{\mu_S} (a, b) = (a + \frac{\mu_S^i}{2} b)^T b \]

\[ \Phi_{\mu_g} (a, b) = \frac{1}{2\mu_g} \left\{ \| \text{Max}(0, a + \mu_g^i b) \|^2 - \| d \|^2 \right\} \]
Non linear Programming Formulation
Augmented Lagrangian with Projection

Karush-Kuhn-Tucker (KKT) first order optimality conditions:

A solution \( x_k, \tau_k, h_k \) to the problem implies there exists Lagrange multipliers \((\lambda_k, \rho_k, \sigma_k)\) and penalty coefficients \(\mu = (\mu_g, \mu_s)\)

such that

\[
\frac{\partial L}{\partial x} = 0, \quad \frac{\partial L}{\partial \tau} = 0, \quad \frac{\partial L}{\partial h} = 0, \quad \frac{\partial L}{\partial \lambda} = 0, \quad \frac{\partial L}{\partial \sigma} = 0, \quad \frac{\partial L}{\partial \rho} = 0,
\]

and

\[\rho_k^T g(x, \tau, h) = 0, \quad \sigma_k^T s(x) = 0, \quad g(x, \tau, h) \leq 0\]

The final state constraint \( x_N = x_T \) is verified in an initial feasible solution. When the control vector is changed, the final state is shifted from the desired one. The adjustment for the final state constraint is done with an orthogonal projection on the tangent space of the constraint:

\[
Pr = I_d - Q^T(QQ^T)^{-1}Q \quad d = - Pr\nabla \tau L\mu
\]
Non linear Programming Formulation

Augmented Lagrangian with Projection

Data Reading

Initial and final states, Lagrange multipliers, State, torques and sampling periods Limits, Tolerances, number of discretizations $N$ and iterations $T^*$

Primal Optimisation

Compute gradients $\nabla_{r_k} L_{\mu}$, $\nabla_{h_k} L_{\mu}$ The co-states backwardly $\lambda_k$

Projection matrix & operator $Q$, $P$ direction descent $d_k$

Update search direction: $d_k = \text{arg Min } L_h(x_k, v_k, \nu + \gamma d_k, \rho, h_k)$

Update control input: $v_{k+1} = v_k - u_k^* d_k$ $h_{k+1} = h_k - u_k^* \nabla L_k$

Update system state: $x_k$

Dual Optimisation

Feasability Test

Cost minimized, Constraints not violated ?

Convergence Test

Cost minimized ? Constraints non violated ? With Optimum Tolerances

Display Optimal Trajectory

$x^*, h^*, v^*$

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Robot and Simulation Parameters

Robot parameters

Robot mass $m = 110$ (Kg), Wheel mass $m_w = 20$ (Kg), Wheel radius $r = 0.25$ (m)

$L = 1.25$ (m) $b = 0.70$ (m) Passage tolerance $T_{\text{PassTh}l} = 10^{-2}$

Limits on workspace, actuator torques, steering angle and sampling periods

<table>
<thead>
<tr>
<th>Parameter</th>
<th>$x$ (m)</th>
<th>$y$ (m)</th>
<th>$\tau_1$ (Nm)</th>
<th>$\tau_2$ (Nm)</th>
<th>$\phi$ (rad)</th>
<th>$h$ (sec)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Max</td>
<td>5</td>
<td>5</td>
<td>10</td>
<td>10</td>
<td>$\pi / 4$</td>
<td>0.1</td>
</tr>
<tr>
<td>Min</td>
<td>-5</td>
<td>-5</td>
<td>-10</td>
<td>-10</td>
<td>$-\pi / 4$</td>
<td>0.001</td>
</tr>
</tbody>
</table>

Augmented Lagrangian parameters

$\nu = 0.1$ $\mu_0 = 0.5$ $\tilde{\nu} = 0.3$ $\gamma_1 = 0.25$ $\gamma_2 = 1.4$ $\omega_s = 0.5$

$\eta_s = 0.5$ $\alpha_w = \alpha_\eta = 0.4$ $\beta_w = \beta_\eta = 0.4$ $\eta^* = \eta^*_l = 10^{-3}$ $\nu^* = 10^{-4}$

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A Scenario Trajectory

From the pose coordinates \((0.1\text{m}, 2\text{m}, 30^\circ)\) to end at the position \((1\text{m}, 5\text{m}, 45^\circ)\),
Initial and final linear and angular velocities are taken to zero \((\text{m/sec})\) and zero \((\text{rad/sec})\),
This trajectory is sampled into a total number of \(N=20\) points.
Implementation on a Wheeled Mobile Robot

A Scenario Trajectory

Three case studies

- Minimum Power
- Minimum Time
- Minimum Time-Power

\[
\begin{align*}
\tau_1 &= 1 \quad \tau_2 = 0 \\
\tau_1 &= 0 \quad \tau_2 = 1 \\
\tau_1 &= 0.5 \quad \tau_2 = 0.5
\end{align*}
\]

Minimum power criterion

Minimum Time criterion

Time-Power criterion

Minimum Time-Power Trajectory is 31% faster than only Minimum Power Trajectory

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The same trajectory with imposed passages over three points (0.2, 2.7), (0.5, 3.4) and (0.8, 2.5) (m).
# Simulation Results

## Convergence history for the simulated trajectory with fixed sampling time and varying sampling time

<table>
<thead>
<tr>
<th>Problem</th>
<th>Fixed sampling time</th>
<th>Varying sampling time</th>
</tr>
</thead>
<tbody>
<tr>
<td>Number of discretizations</td>
<td>20</td>
<td>20</td>
</tr>
<tr>
<td>Number of Inner Iterations</td>
<td>8</td>
<td>8</td>
</tr>
<tr>
<td>Number of Outer Iterations</td>
<td>7</td>
<td>7</td>
</tr>
<tr>
<td>Number of Adjustments</td>
<td>14</td>
<td>23</td>
</tr>
<tr>
<td>Travelling Time (sec)</td>
<td>2</td>
<td>1.31</td>
</tr>
<tr>
<td>PREC (m)</td>
<td>$0.24 \times 10^{-4}$</td>
<td>$0.45 \times 10^{-4}$</td>
</tr>
<tr>
<td>CPU (sec)</td>
<td>178</td>
<td>209</td>
</tr>
<tr>
<td>Power (W)</td>
<td>167</td>
<td>224</td>
</tr>
</tbody>
</table>
Conclusions and Discussions

The Problem

- Multi-Objective Trajectory Planning for Wheeled Mobile Robots
- A Non-Linear and Non-Convex Constrained Optimal Control Problem

Augmented Lagrangain with Projection (ALP) allows

- Giving smooth with monotonous increasing energy trajectories
- Kinematic solution feasible, but… torques Outside the admissible domain
- Minimum Time-Power 31% faster than only minimum power criterion
- Implement a Gradient Projection to reach the final state at each iteration
Conclusions and Discussions

Limitations & Ways for Improvement

- Achieve robustness test by changing dynamic parameters such as inertia
- Implement a Mixed Integer-Non-Linear Program for Time Minimization
- Achieve experimentation and measurements with a physical robot

Perspectives and Future Trends

- Use the outcomes of this offline trajectory planning system along with physical measurements:
  - as reference trajectory to build a feedback control system for adaptive multi-objective online planning
  - as dataset trajectories to build an adaptive neuro-fuzzy control system for online multi-objective planning

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Thank you for your attention!!