B.4 Problem Set IV

(Due Wed. Nov. 4)[4]

Problem B.4.1. Compute numerically the vector J_μ satisfying

$$J_\mu = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix} + \alpha \begin{bmatrix} 3/4 & 1/4 & 0 \\ 1/4 & 3/4 - \epsilon & \epsilon \\ 0 & \epsilon & 1 - \epsilon \end{bmatrix} J_\mu,$$

for $\alpha = 0.9$ and $\alpha = 0.999$ and $\epsilon = 0.5$ and $\epsilon = 0.001$. For this, try value iteration with and without error bounds, as well as Gauss-Seidel value iteration. Discuss your results.

Problem B.4.2. Policy iteration is related to Newton's method to solve nonlinear equations. Consider an equation of the form $F(J) = 0$, where $F : \mathbb{R}^n \to \mathbb{R}^n$. Given a vector $J_k \in \mathbb{R}^n$, Newton's method determines J_{k+1} by solving the linear system of equations (coming from the first-order Taylor expansion of F)

$$F(J_k) + \frac{\partial F(J_k)}{\partial J}(J_{k+1} - J_k) = 0,$$

where $\frac{\partial F(J_k)}{\partial J}$ is the Jacobian matrix of F evaluated at J_k.

1. Consider the discounted finite-state problem and define $F(J) = TJ - J$. Show that if there is a unique μ such that $T_\mu J = TJ$, then the Jacobian matrix of F at J is
$$\frac{\partial F(J)}{\partial J} = \alpha P_\mu - I,$$
where I is the $n \times n$ identity matrix.

2. Show that the policy iteration algorithm can be identified with Newton's method for solving $F(J) = 0$, assuming it gives a unique policy at each step.

Problem B.4.3. [Bertsekas] $1.14 - a$).

[4] this version: Oct. 20 2009