Geometric Programming and Mechanism Design for Air Traffic Conflict Resolution

Jerome Le Ny and George J. Pappas

***Abstract*— We develop certain extensions of optimization-based conflict resolution methods in air traffic control. The problem considered concerns the scheduling of the crossing times of a set of aircraft through a metering fix, while maintaining aircraft separation. First, we show how to solve this combined path planning and scheduling problem using mixed-integer *geometric* programming. Second, the objective function used to determine the aircraft ordering at the fix is not given a priori but needs to be obtained from the airlines, which are strategic profit maximizing agents and could lie about their true cost. In order to realign individual and global objectives, we study the use of the Clarke-Groves mechanism in this context, which aims at extracting the true utility functions from the airlines using side-payments to the FAA.**

I. Introduction and Motivation

With the growth of airspace congestion, there has been significant research activity in the past decade to develop automated systems for air traffic conflict detection and resolution. Conflict resolution, i.e., the local modification of aircraft trajectories in order to maintain mandatory safety separation distances, is usually studied solely from the point of view of the air traffic controller (ATC). In this setting, numerous conflict resolution methods have been proposed, such as predefined resolution maneuvers or optimization based approaches [1]. In the latter case, the cost metrics considered for optimization typically include fuel, time and potentially ATC workload. These metrics however do not necessarily lead to good solutions for the airlines and the passengers. For example, a plane with passengers trying to catch a connecting flight incurs a much greater penalty for an additional conflict resolution delay than a plane that is ahead of schedule. Thus depending on the circumstances, the airlines have different valuations for the additional delays imposed by the ATC, and might be willing to negotiate a solution more compatible with their individual preferences. Their valuations however are private and not available to the ATC for optimizing the trajectories. This is an important aspect of the problem because simply polling the airlines for their valuations of delays lacks any incentive for them to communicate truthfully. A more carefully designed mechanism is necessary in order to prevent them from exaggerating the importance of the penalties imposed by additional delays.

Mechanism design [2], [3] is concerned with the design of institutions for collective decision making despite agents pursuing their individual interests. This topic has in recent years been the subject of an increasing interest for designing engineered systems, in particular communication and computer networks, see e.g. [4]. It forms the foundation of auction theory, and as such it has been used in the design of auctions and trading mechanisms for the allocation of arrival and departure slots at airports during Ground Delay Programs [5], [6]. Moreover, the concept of "Collaborative Decision Making" (CDM) [7] adopted by the Federal Aviation Administration (FAA) emphasizes the idea that decisions which have a potential economic impact on the airlines should be made in collaboration with them. This paradigm leaves room for a wide range of applications of mechanism design for air traffic flow management, see e.g. [8].

In this paper we propose the application of mechanism design to en route conflict resolution between multiple aircraft. With respect to previous work on optimization-based approaches to this problem [9]–[12], the motion planning problem considered here is fairly simple. However we assume that the airlines can lie about the cost function that is ultimately used for optimization. Hence it is necessary to design an additional mechanism in order to extract the true utility function of the airlines, and for this purpose we study the Clarke-Groves mechanism. An additional extension of the previous work is that we consider a joint motion planning and scheduling problem (for aircraft moving towards an airspace fix), for which standard linear programming [9] and mixed-integer linear programming formulations [12], or even relaxations based on quadratically constrained quadratic programs [10] are not appropriate. Instead, we solve our problem using a mixed-integer geometric programming formulation, which is of independent interest.

The rest of the paper is organized as follows. Section II presents the problem formulation, and describes the constraints of the optimization problem that result from the safety conditions (aircraft separation). We also include some background material on geometric programming. In section III we present the Clarke-Groves mechanism and show how it can be used to extract the cost functions of the airlines using side payments. Section IV summarizes the solution procedure, and section V presents simulation results. Finally, we conclude in section VI and briefly discuss some research directions left for future work.

II. Problem Formulation

We consider a scenario (see Fig. 1) where N aircraft are initially in configurations $(\hat{x}_i, \hat{y}_i, \psi_i) \in \mathbb{R}^2 \times \mathcal{S}^1, i =$

This work was supported by the ONR-MURI award N00014-08-1-0696.

The authors are with the Department of Electrical and Systems Engineering, University of Pennsylvania, Philadelphia, PA 19104, USA jeromel@seas.upenn.edu, pappasg@seas.upenn.edu.

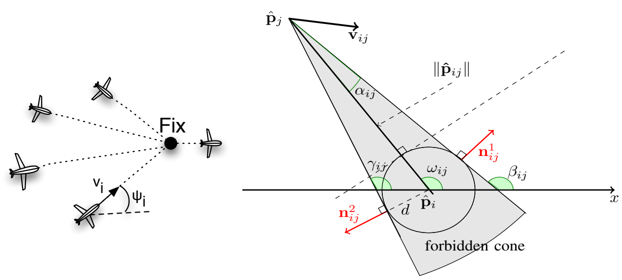

Fig. 1. Left: aircraft must merge at the airspace fix. Separation is ensured after the fix. Right: geometric representation of the forbidden cone of velocities.

$1, \ldots, N$, and have initial velocity vectors $\hat{\mathbf{v}}_i \in \mathbb{R}^2$ (throughout the paper, we denote vector quantities in boldface). Hence the orientation of $\hat{\mathbf{v}}_i$ with respect to a fixed horizontal axis is ψ_i, and we let $\hat{v}_i = \|\hat{\mathbf{v}}_i\|$. Also, let $\hat{\mathbf{p}}_i = (\hat{x}_i, \hat{y}_i)$ denote the initial position of aircraft i. The problem considered is two-dimensional. There is a metering fix or merging point with known position through which all the aircraft must pass, subject to a Minutes-In-Trail restriction MinIT. This means that two successive aircraft passing through the point must be separated in time by at least MinIT minutes. The mandatory separation distance at all times between aircraft is d. We initially assume the aircraft to be separated. Moreover, we remove an aircraft from consideration once it passes through the fix, since typically such points are used to generate downstream a single flow of separated aircraft with identical headings, for example in preparation for landing at an airport. The initial headings ψ_i are assumed to be directed towards the fix and cannot be changed. The only control available to the ATC is a single change in the magnitude of the velocity vector for each aircraft, which is executed instantaneously at time $t = 0$, a problem considered in [12]. After the change imposed by the ATC, the speed of aircraft i becomes v_i, subject to the constraints

$$0 < v_{i,min} \leq v_i \leq v_{i,max} \tag{1}$$

imposed for passenger comfort as well as physical reasons, such as stall and Mach buffeting [10]. The variables v_i serve as decision variables in the formulation of the optimization problem. Hence the velocity vectors are fixed to $\mathbf{v}_i, i = 1, \ldots, N$, with $\|\mathbf{v}_i\| = v_i$ once the initial change in magnitude has been executed.

A. Separation Constraints

The fact that any two aircraft must be separated by the distance d at all times translates into constraints imposed on the allowed velocities. Consider two aircraft i, j with velocities $\mathbf{v}_i, \mathbf{v}_j$, positions $\mathbf{p}_i, \mathbf{p}_j$, and initial positions $\hat{\mathbf{p}}_i, \hat{\mathbf{p}}_j$ (see Fig. 1). We study the problem in the mobile frame centered at $\mathbf{p}_i$. In this frame, aircraft j has relative velocity $\mathbf{v}_{ij} = \mathbf{v}_j - \mathbf{v}_i$ and aircraft i is immobile. We first impose a safety condition considered in previous work [10], [12]. No conflict arises if the distance from $\hat{\mathbf{p}}_i$ to the half-line $\hat{\mathbf{p}}_j + \mathbb{R}_+ \mathbf{v}_{ij}$ describing the trajectory of j in the moving frame is at least d. Geometrically, this means that the velocity vector $\mathbf{v}_{ij}$ lies outside of a "forbidden convex cone" with apex at $\hat{\mathbf{p}}_j$ and tangent to the disc centered at $\hat{\mathbf{p}}_i$ and of radius d (see Fig. 1). This constraint has been handled in previous work in at least two ways. In [10], it is represented by a nonconvex quadratic inequality on $\|\mathbf{v}_{ij}\|$ in the optimization problem. In the solver, this constraint is relaxed to obtain a semidefinite program, the solution of which serves to design a feasible solution using randomized rounding. We follow an alternative approach, as in [12], which consists in representing the forbidden cone as the intersection of two half-spaces defined by two normal vectors $\mathbf{n}^1_{ij}$ and $\mathbf{n}^2_{ij}$ such that the *admissible* relative velocities satisfy

$$\left[\langle \mathbf{v}_j - \mathbf{v}_i, \mathbf{n}^1_{ij} \rangle \geq 0\right] \quad \vee \quad \left[\langle \mathbf{v}_j - \mathbf{v}_i, \mathbf{n}^2_{ij} \rangle \geq 0\right], \tag{2}$$

where $\vee$ denotes the logical "or" operator (see Fig. 1). This separation constraint can be formulated as a disjunction of linear constraints on the decision variables v_i, v_j (see [12] and subsection II-C).

Condition (2) guarantees safety over an *infinite horizon* in the case where the velocities are never changed after $t = 0$. In particular, it does not allow an aircraft following another aircraft and moving in the same direction to catch up with the preceding aircraft, because eventually a conflict would arise. This condition is too conservative in our metering application, where separation between two aircraft must be ensured only until one them reaches the fix, since we assume that safety is guaranteed beyond it by a new set of ATC commands. Hence we wish to allow for two aircraft following each other to reduce their separation, potentially up to the minimum imposed by the MinIT restriction. We consider a sufficient condition ensuring that separation is maintained until one of the aircraft reaches the position of the airspace fix. Let d_i be the initial distance (i.e., at $t = 0$) from $\hat{\mathbf{p}}_i$ to the position of the fix. The time t_i it takes to aircraft i to reach the fix is $t_i = d_i / v_i$. Referring to Fig. 1, safety is maintained if aircraft i reaches the fix position before j enters the circle around i, or if j reaches the fix position before i enters the similar circle around j. Let us consider the first case, and note on Fig. 1 the dashed line perpendicular to $\hat{\mathbf{p}}_{ij} = \hat{\mathbf{p}}_j - \hat{\mathbf{p}}_i$ tangent to the circle around aircraft i, which separates the plane into two half-planes, each containing one aircraft. The sufficient condition that we consider consists in allowing cases where $\mathbf{v}_{ij}$ belongs to the forbidden cone, i.e., condition (2) is not satisfied, as long as aircraft j remains on the side of the plane that does not contain i until time t_i. It is only a sufficient safety guarantee because it ignores the fact that safety could also be maintained even if j were going past this line into the side regions of the forbidden cone around the circle. This safety condition can be written $\langle \hat{\mathbf{p}}_{ij}, \hat{\mathbf{p}}_{ij} + \frac{d_i}{v_i} \mathbf{v}_{ij} \rangle \geq d \|\hat{\mathbf{p}}_{ij}\|$. The symmetric constraint with the roles of i and j are inverted can be written $\langle \hat{\mathbf{p}}_{ij}, \hat{\mathbf{p}}_{ij} + \frac{d_j}{v_j} \mathbf{v}_{ij} \rangle \geq d \|\hat{\mathbf{p}}_{ij}\|$. If *either* of these conditions is verified, then safety is maintained. Indeed suppose the first condition is satisfied but not the second. Then, because we have $\langle \hat{\mathbf{p}}_{ij}, \mathbf{v}_{ij} \rangle < 0$ when $\mathbf{v}_{ij}$ belongs to the forbidden cone, this implies $t_i < t_j$. Hence safety is ensured because i crosses the fix first. These constraints can be again transformed into linear constraints on the variables v_i, v_j, but we will not use this fact. Finally, our separation constraints can be summarized with the following disjunctions, one for each unordered pair $i \neq j$

$$\begin{aligned} &[\langle \mathbf{v}_j - \mathbf{v}_i, \mathbf{n}_{ij}^1 \rangle \geq 0] \quad \vee \quad [\langle \mathbf{v}_j - \mathbf{v}_i, \mathbf{n}_{ij}^2 \rangle \geq 0] \\ \vee &\left[\langle \hat{\mathbf{p}}_{ij}, \hat{\mathbf{p}}_{ij} + \frac{d_i}{v_i}(\mathbf{v}_j - \mathbf{v}_i) \rangle \geq d \|\hat{\mathbf{p}}_{ij}\| \right] \\ \vee &\left[\langle \hat{\mathbf{p}}_{ij}, \hat{\mathbf{p}}_{ij} + \frac{d_j}{v_j}(\mathbf{v}_j - \mathbf{v}_i) \rangle \geq d \|\hat{\mathbf{p}}_{ij}\| \right]. \end{aligned} \tag{3}$$

B. Scheduling Constraints due to Minutes-in-Trail Restrictions

Besides the *separation constraints*, we have constraints originating from the minutes-in-trail restriction, which we call *scheduling constraints*. We have, for each pair $i \neq j$:

$$\left[\frac{d_i}{v_i} - \frac{d_j}{v_j} \geq \mathsf{MinIT} \right] \vee \left[\frac{d_j}{v_j} - \frac{d_i}{v_i} \geq \mathsf{MinIT} \right]. \tag{4}$$

The first constraint must be satisfied if aircraft j passes through the fix before aircraft i, and the second constraint if the order is reversed. Unfortunately, the constraints inside the logical operator are not linear in the decision variables $\{v_i\}_{1 \leq i \leq N}$, hence cannot be handled, at least directly, by a linear programming solver as in [12]. As we discuss next, they can be handled by a geometric programming solver however, which is almost as efficient.

C. Geometric Programming

The basic property that we rely on for solving reasonably fast optimization problems subject to mixed continuous and logical constraints is that we have a solver able to solve optimization problems for an arbitrary conjunction of the atomic constraints appearing in the disjunctions. This solver is used in any method proposed to solve the global optimization problem, for example to obtain the required bounds in a branch-and-bound method. In the MILP approach used in [12], this solver is a linear programming solver. Due to the nonlinearity of our scheduling constraints (4), we replace it here by a geometric programming (GP) solver. This requires that we convert all disjuncts appearing in the constraints to constraints which can be handled by this GP solver. First, we review the basic terminology of geometric programs, see e.g. [13]. A *monomial* is a function $f : \mathbb{R}^n_{>0} \to \mathbb{R}$ of the form $f(x) = c x_1^{a_1} x_2^{a_2} \cdots x_n^{a_n}$, where $c > 0$ and $a_i \in \mathbb{R}$, for $1 \leq i \leq n$. A function f which is the sum of one or more monomials $f(x) = \sum_{k=1}^K c_k x_1^{a_{1k}} x_2^{a_{2k}} \cdots x_n^{a_{nk}}$, is called a *posynomial*. Optimization problems involving the minimization of a posynomial function subject to constraints of the form $g(x) \leq 1$ with g a posynomial as well as $h(x) = 1$ with h a monomial, are called *geometric programs* (GP), and can be solved almost as efficiently as linear programs by interior-point methods [13]. Next, a *generalized posynomial* is a function that can be formed from posynomials using the operations of addition, multiplication, positive fractional power, and maximum. Replacing posynomials by generalized posynomials in the definition of a geometric program, we obtain a *generalized geometric program*, which can be solved efficiently as well by converting it to a GP.

Note that the scheduling constraint $\frac{d_i}{v_i} - \frac{d_j}{v_j} \geq \mathsf{MinIT}$ can be rewritten $\frac{\mathsf{MinIT}}{d_i} v_i + \frac{d_j}{d_i} v_i \, v_j^{-1} \leq 1$, hence is of the form $g(v_i, v_j) \leq 1$, where g is a posynomial. For the separation constraints, we will use the following lemma.

Lemma 1: Any linear constraint of the form

$$\alpha x + \beta y \geq 0, \quad \text{with } x > 0, y > 0 \tag{5}$$

is either trivial (if $\alpha, \beta \geq 0$), infeasible (if $\alpha, \beta \leq 0$ and $\alpha + \beta \neq 0$), or can be rewritten as $g(x, y) \leq 1$, where g is a posynomial.

Proof: The last case is when α and β have opposite signs and are both non-zero. If $\alpha > 0, \beta < 0$, then we can rewrite the inequality as $\frac{|\beta|}{\alpha} \frac{y}{x} \leq 1$. If $\alpha < 0, \beta > 0$, we rewrite the constraint as $\frac{|\alpha|}{\beta} \frac{x}{y} \leq 1$. ∎

We see that by a simple preprocessing of the linear inequalities of the form (5), we can handle such inequalities by geometric programming. Since the constraints will appear in a disjunction of constraints, the preprocessing consists in removing the infeasible constraints ($\alpha \leq 0, \beta \leq 0, \alpha + \beta \neq 0$) from the disjunction, and removing the disjunction altogether if a trivial constraint is found ($\alpha \geq 0, \beta \geq 0$), since in the latter case the disjunction is a tautology.

We now write the separation constraints (3) in coordinates. Let $\alpha_{ij} = \arcsin(d / \|\hat{\mathbf{p}}_{ij}\|)$ (then $\alpha_{ij} \in (0, \pi/2]$), and define with respect to the global coordinate system $\omega_{ij} = \arg(\hat{\mathbf{p}}_{ij})$, $\beta_{ij} = \omega_{ij} + \alpha_{ij}$ and $\gamma_{ij} = \omega_{ij} - \alpha_{ij}$ (see Fig. 1). Then in coordinates we have $\mathbf{n}_{ij}^1 = [\sin \beta_{ij}, -\cos \beta_{ij}]^T$ and $\mathbf{n}_{ij}^2 = [-\sin \gamma_{ij}, \cos \gamma_{ij}]^T$. Hence the first constraint in the disjunction becomes

$$\begin{aligned} &v_i(\sin \psi_i \cos \beta_{ij} - \cos \psi_i \sin \beta_{ij}) \\ &\quad + v_j(\cos \psi_j \sin \beta_{ij} - \sin \psi_j \cos \beta_{ij}) \geq 0. \end{aligned}$$

For the second constraint in (3), we get similarly

$$\begin{aligned} &v_i(\cos \psi_i \sin \gamma_{ij} - \sin \psi_i \cos \gamma_{ij}) \\ &+ v_j(\sin \psi_j \cos \gamma_{ij} - \cos \psi_j \sin \gamma_{ij}) \geq 0. \end{aligned}$$

The third inequality is

$$\begin{aligned} &v_i \|\hat{\mathbf{p}}_{ij}\| (\|\hat{\mathbf{p}}_{ij}\| - d - d_i \cos \omega_{ij} \cos \psi_i - d_i \sin \omega_{ij} \sin \psi_i) \\ &\quad + v_j \|\hat{\mathbf{p}}_{ij}\| d_i (\cos \omega_{ij} \cos \psi_j + \sin \omega_{ij} \sin \psi_j) \geq 0. \end{aligned}$$

Finally, the last inequality is

$$\begin{aligned} &v_j \|\hat{\mathbf{p}}_{ij}\| (\|\hat{\mathbf{p}}_{ij}\| - d + d_j \cos \omega_{ij} \cos \psi_j + d_j \sin \omega_{ij} \sin \psi_j) \\ &\quad - v_i \|\hat{\mathbf{p}}_{ij}\| d_j (\cos \omega_{ij} \cos \psi_i + \sin \omega_{ij} \sin \psi_i) \geq 0. \end{aligned}$$

All of these four inequalities are of the form (5), so feasible and non-trivial disjuncts can be handled by a GP solver.

In conclusion, each of the literals in (3), (4) is a geometric programming constraint. In the previous work [12] which included disjunctions of linear constraints only, the resulting mixed logical continuous optimization problem was solved using a mixed integer linear programming (MILP) solver and the "big-M" method to model the disjunction, see section IV.

MILP cannot handle the scheduling constraints, but we will see how to solve an optimization problem with mixed logical and geometric constraints using mixed-integer geometric programming in section IV. Interestingly, the standard big-M method used for MILP must be modified to work with a geometric programming solver.

III. Extracting User Preferences

A. Parameterization of the Cost Functions

Previous work on optimization-based methods for conflict resolution in air traffic control has assumed the objective function to be given or defined by the optimizer (i.e., the ATC), see e.g. [9], [10], [12], [14], [15]. However, ultimately the goal of the system once aircraft separation is ensured by the ATC is to optimize the preferences of the airlines, since this has a more direct impact on the travelers. At any given time if a conflict occurs, different planes might have different tolerances to additional delays. Whereas some might be trying to make a connection, others might be willing to accept a larger deviation from their nominal path against, say, a monetary reward. However in a strategic environment it is not possible to simply ask the airlines to communicate their cost functions in the absence of any incentive for them to do so truthfully. In this section, we describe a mechanism that can be implemented by the ATC in order to extract the cost functions from the aircraft. The sum of these cost functions is then minimized (as an aggregate of the preferences of the aircraft involved in the conflict), subject to the safety and scheduling constraints described in section II.

We assume that the preference of each aircraft involved in the conflict (as determined by the individual airlines) is expressed in monetary units by a cost function of v_i belonging to a family of *generalized posynomials* (see section II-C) with a finite dimensional parameterization by a set Θ_i. For example, we will assume in the simulations of this paper that this cost function is a posynomial function of v_i of the form:

$$f(v_i;\theta_i) = c_{0,i} + c_{1,i}\, v_i^{a_{i,1}} + \ldots + c_{K,i}\, v_i^{a_{K,i}},$$

where $c_{i,j} \geq 0$ and $a_{i,j} \in \mathbb{R}$ for all $i = 1,\ldots,N, j = 1,\ldots,K$ (so here $\Theta_i = \mathbb{R}_+^{K+1} \times \mathbb{R}^K$). Here the maximum number of terms K is fixed and the same for all aircraft. We define the parameter $\theta_i = [c_{0,i}, c_{1,i}, \ldots, c_{K,i}, a_{1,i}, \ldots, a_{K,i}]$, called the *type* of aircraft i. For our application, a typical shape of the cost function might include a minimum cost at a preferred velocity, for example at $\hat{v}_i$, and increasing cost for large velocities due to fuel consumption and for low velocities due to additional delays ([13] described techniques that can be used by the airlines to approximate their real cost function by a generalized posynomial). The parameter θ_i controlling the shape of the cost function of an airline is not known to the ATC, and airline would not communicate their type truthfully without proper incentive.

B. Mechanism Design

We now describe a system which aims at correcting this problem, based on principles developed in the *mechanism design* literature [2], [3]. As before, the ATC asks the aircraft to send their parameters $\{\theta_i\}_{1\leq i\leq N}$. We assume for simplicity that these aircraft *do not share their type information with each other*, for instance because they belong to different airlines. Aircraft i sends a value $\hat{\theta}_i$ to the ATC, which might a priori differ from its true type θ_i. The ATC then simply optimizes the sum of the cost functions $\sum_{i=1}^N f(v_i;\hat{\theta}_i)$ subject to the scheduling and separation constraints of section II. We add, however, an additional mechanism to ensure that the aircraft announce a correct parameter, essentially by using side-payments for penalizing the announcement of "larger" cost functions. Denote by $\Theta = \prod_{i=1}^N \Theta_i$ the cartesian product of all type sets. Let $\mathsf{V} \subset \mathbb{R}_+^N$ be the feasible set of velocities satisfying the constraints described in section II. Redefine the cost function f of aircraft i defined earlier as a function $f_i : \mathsf{V} \times \Theta_i \to \mathbb{R}$, i.e. $f_i(v,\theta_i) \equiv f(v_i;\theta_i)$. The action of the ATC, deciding a new speed for each aircraft based on the announced preferences, can then be described by a decision rule $d : \Theta \to \mathsf{V}$. Such a rule is called *efficient* if it minimizes the total cost

$$\sum_{i=1}^N f_i(d(\theta),\theta_i) \leq \sum_{i=1}^N f_i(d',\theta_i),\ \forall\theta,\ \forall d' \in \mathsf{V}. \tag{6}$$

To align the objectives of the various airlines, it is necessary to use transfers (say monetary) among them or between them and the ATC, prescribed by a transfer function $t : \Theta \to \mathbb{R}^N$. Here $t_i(\hat{\theta})$ represents a payment that aircraft i makes (or receives if it is negative) to the ATC based on the announcement of the types $\hat{\theta}$ by all aircraft.

The pair of functions (d,t) is referred to as a *social choice function* [3][1], because it implements a social choice by aggregating individual preferences. The total cost for aircraft i, if $\hat{\theta}$ is the vector of announced types, i's true type is θ_i, and the social choice function is (d,t), is

$$\tilde{f}_i(\hat{\theta},\theta_i,d,t) = f_i(d(\hat{\theta}),\theta_i) + t_i(\hat{\theta}).$$

A transfer function t is said to be *feasible* if $\sum_{i=1}^N t_i(\theta) \geq 0$ for all θ, i.e., no transfer is made into the system from an outside source. Note that the ATC does not take into account the transfer function t when computing an efficient decision d according to (6). Hence if $\sum_i t_i > 0$, then there is some net loss in utility to the players as a whole relative to an efficient decision with no transfer. In this case, we have a surplus that cannot be returned to the airlines, otherwise the transfer function and incentives would be modified.

The data of the parameter space Θ and the social choice function (d,t), which are communicated in advance and thus known to the aircraft, constitutes what is called a (direct) *mechanism* [3]. With the specification of the family of cost functions as above, a mechanism induces a game between the players, whose strategies consist in selecting which parameter $\hat{\theta}_i$ they should announce to the ATC (hence the strategy space of aircraft i is simply Θ_i). One objective of mechanism design, among others, is to design the social

[1] In the economics literature, one usually maximizes a utility function instead of minimizing a cost, and the transfer function t is usually taken to be positive if a payment is received

choice function in order to ensure that the players announce their true type θ_i, so that the ATC performs the optimization with the correct cost function. Even with the addition of the transfer payments t to the ATC, this scheme might still result in a cost for the aircraft that is better in general than a scheme assuming a priori an objective function which does not take into account the individual preferences of the airlines (see the simulations in section V).

For a vector $\theta = (\theta_1, \ldots, \theta_n)$, we define $\theta_{-i} = (\theta_1, \ldots, , \theta_{i-1}, \theta_{i+1}, \ldots, \theta_n)$. In order to predict how the players interacting through the mechanism behave, we consider the following notion

Definition 2: A strategy $\hat{\theta}_i \in \Theta_i$ is a *dominant strategy* at $\theta_i \in \Theta_i$ if

$$\tilde{f}((\hat{\theta}_i, \hat{\theta}_{-i}), \theta_i, d, t) \leq \tilde{f}((\tilde{\theta}_i, \hat{\theta}_{-i}), \theta_i, d, t)$$

for all $\hat{\theta}_{-i}$ and $\tilde{\theta}_i$.

Hence a dominant strategy for player i is optimal (taking into account the payment) no matter what the other players do under the announced mechanism, and is therefore a good candidate for the expected behavior of the players if it exists. A direct mechanism (d, t) is *dominant strategy incentive compatible* if θ_i is a dominant strategy at θ_i for each i and $\theta_i \in \Theta_i$. That is, under such a mechanism, *it is always a dominant strategy for a player to announce his true type*. In this paper, we will consider only a simple mechanism that has nice properties, and was described initially in the work of Clarke [16] and Groves [17].

Definition 3: The *pivotal mechanism* for the conflict resolution problem consists in

- an efficient decision rule (6), i.e., the ATC minimizes the total cost based on the types announced by the aircraft.
- a transfer function consisting in a payment

$$t_i(\hat{\theta}) = \sum_{j \neq i} f_j(d(\hat{\theta}), \hat{\theta}_j) - \left(\min_{v \in \mathsf{V}} \sum_{j \neq i} f_j(v, \hat{\theta}_j) \right) \quad (7)$$

  for each aircraft $i \in \{1, \ldots, N\}$ (the payment amounts (7) are computed by the ATC).

Note that the transfer is always non-negative and hence is feasible. Note also that it has the nice property that if the presence of i makes no difference in the minimizing decision v for the other aircraft, then $t_i(\hat{\theta}) = 0$. Hence a player always has the possibility of announcing a cost function of 0 showing indifference to any delay in order to avoid any payment. However, this choice would not be optimal in general, because of the following result.

Theorem 4 (see e.g. [3]): The pivotal mechanism is dominant strategy incentive compatible.

Hence under this mechanism, the players should announce their true type. From (7), we can see that the payment of aircraft i represents the penalty incurred by the other aircraft due to the change in decision that results from i's presence.

IV. Solving the Optimization Problem

Using the pivotal mechanism, we can now assume that the ATC has a cost function at his disposal in order to make a decision $v = (v_1, \ldots, v_N) \in \mathsf{V}$. Recall that after receiving the parameter values $\hat{\theta}_1, \ldots, \hat{\theta}_N$ from each aircraft, the ATC needs to minimize the objective $\sum_{i=1}^N f_i(v, \hat{\theta}_i)$ over $v \in \mathsf{V}$, subject to the velocity bounds (1), the separation constraints (3) and the scheduling constraints (4). In the following we discuss how this optimization problem can be solved as a mixed-integer geometric program (MIGP), e.g. using YALMIP [18]. Recall the standard "big-M" formulation used to model a disjunction of linear constraint $a_1^T x \leq b_1 \vee a_2^T x \leq b_2$, by introducing an integer variable. Assuming we have bounds on the variables x, we can rewrite the disjunction above as the conjunction

$$a_1^T x \leq b_1 + cM, a_2^T x \leq b_2 + (1-c)M, c \in \{0, 1\},$$

for some sufficiently large M. A mixed-integer linear programming (MILP) solver using branch-and-bound obtains lower bounds by relaxing the binary constraint to $0 \leq c \leq 1$. This formulation does not work directly with geometric programming however, because the equivalent of the first constraint would be $f(x) \leq 1 + cM$, with f a posynomial, which is *not* a geometric programming constraint when c is relaxed to $0 \leq c \leq 1$ (since $f(x) - cM$ is not a posynomial). We can use instead the following modification of the method. Consider the conjunction

$$f(x) + 2M/c \leq 1 + 2M, g(x) + cM \leq 1 + 2M, c \in \{1, 2\},$$

for M sufficiently large (assuming we have bounds $0 < \underline{b} < x < \overline{b}$, as in our problem). Then for $c = 1$, the first constraint is enforced, and for $c = 2$, the second constraint is enforced. When the constraint on c is relaxed to $1 \leq c \leq 2$, we have a standard geometric program.

More generally, for a disjunction of n posynomial constraints $f_1(x) \leq 1 \vee \ldots \vee f_n(x) \leq 1$ we can introduce n integer variables $b_1, \ldots, b_n \in \{1, 2\}$ and consider the conjunction of constraints $f_i(x) + b_i M \leq 1 + 2M$, and in addition the posynomial constraint $2/(b_1 \ldots b_n) \leq 1$, which forces at least one of the $b_i's$ to be 2, and the corresponding constraint in the disjunction to be enforced. Modeling the disjunctions in the separation and scheduling constraints this way, we obtain a mixed integer geometric program.

V. Simulations

In this section we illustrate the distribution of payments according to the pivotal mechanism in a particular scenario. Between 2 and 4 aircraft are generated with random positions in a 100×100 nm square, which must all pass through an airspace fix situated 400 nm away as depicted on Fig. 1. The velocity of each aircraft is bounded between 350 and 450 kn. The initial conditions correspond to the time at which the conflict is detected and the new velocities are then assigned by the ATC. A Minutes-In-Trail restriction of 2 minutes between successive aircraft is enforced at the fix. The aircraft cost functions are shown on Fig. 2. The same functions are used in all experiments, only the initial positions of the aircraft change. In experiments with i aircraft, we use the cost functions $1, \ldots, i$, for $i = 2, 3, 4$. There are cases where the optimization problem is infeasible, i.e., the ATC cannot

TABLE I

FINAL RELATIVE PAYMENTS.

	Scenario 1	Scenario 2	Scenario 3
Mean	0.03%	0.35%	2.64%
Std Dev.	0.29	2.09	5.83
Max	3.49%	15.5%	34.1%
% cases with payments	3	9	26

separate the aircraft using only velocity changes. These cases were excluded from the simulation results presented.

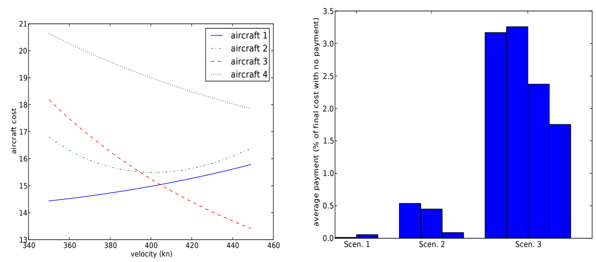

Fig. 2. Left: cost functions of the individual aircraft. Right: final relative payments $t_i/f_i(v_i)$ of the aircraft under ATC assigned velocities v. For each scenario (with $2, 3$ and 4 aircraft), averages are taken over 100 simulations.

We performed 100 simulations in each of 3 scenarios, involving 2, 3 and 4 aircraft respectively. The final average relative aircraft payments $t_i/f_i(v)$ under the velocities assigned by the ATC are shown on Fig. 2. The relatively low mean payments are due to the fact that in a large number of cases, the solver could separate the aircraft while assigning them their preferred velocities, resulting in no payments. There are however large variations in payment amounts in the cases where a conflict arises which requires deviations from preferred velocities, see Table I. In Scenario 2 with 3 aircraft, in some cases aircraft 2 could pay up to an additional 15% of the cost it already incurs at the assigned velocity, whereas in Scenario 3 with 4 aircraft, this number climbed up to 34% for aircraft 1. The higher average payments of aircraft 1 and 2 are probably due to the fact that is is more difficult to accommodate their preference for a lower speed, since with our limited controls this requires slowing down the aircraft situated behind them as well.

VI. CONCLUSIONS AND FUTURE WORK

We have proposed to apply some mechanism design principles in order to resolve conflicts in air traffic control at the tactical level. The mechanism can allow airlines to express their sensitivities to additional delays. Future work will investigate the extensions to situations with more ATC actuation capabilities, such as heading change, vector for spacing, etc. At the same time, it is important to understand the impact of the payments on the airline costs and willingness to participate in such a scheme. The simple scenarios presented in this paper suggest that on average the payments can be kept quite low, with wide fluctuations when aircraft separation requires large trajectory deviations. There are also other mechanisms of interest, in particular if one removes the requirement of dominant strategy incentive compatibility [3]. Such schemes in some settings can for example overcome the balance difficulties exhibited by the pivotal mechanism (i.e., the fact that $\sum_i t_i > 0$).

Finally, the particular scenario studied in this paper is a joint scheduling and path planning problem, extending previous works that only consider the path planning component. Geometric programming proved to be useful in this context, and we have shown how to model disjunctions with a MIGP solver. We are currently working on a more efficient solver for such mixed logic convex programs.

REFERENCES

[1] J. Kuchar and L. Yang, "A review of conflict detection and resolution modeling methods," *IEEE Transactions on Intelligent Transportation Systems*, vol. 1, no. 4, pp. 179–189, 2000.

[2] A. Mas-Colell, M. Whinston, and J. Green, *Microeconomic Theory*. Oxford University Press, 1995.

[3] M. Jackson, "Mechanism theory," Stanford University, Tech. Rep., 2003.

[4] N. Nisan, T. Roughgarden, E. Tardos, and V. Vazirani, Eds., *Algorithmic Game Theory*. Cambridge University Press, 2007.

[5] T. Vossen, "Fair allocatio methods in air traffic management," Ph.D. dissertation, University of Maryland, 2002.

[6] T. Vossen and M. Ball, "Slot trading opportunities in collaborative ground delay programs," *Transportation Science*, vol. 40, no. 1, pp. 29–43, February 2006.

[7] Collaborative Decision Making Webpage. [Online]. Available: http://cdm.fly.faa.gov/

[8] S. Waslander, "Multi-agent systems design for aerospace applications," Ph.D. dissertation, Stanford University, 2007.

[9] W. Niedringhaus, "Stream option manager (SOM): Automated integration of aircraft separation, merging, stream management, and other air traffic control functions," *IEEE Transactions on Systems, Man, and Cybernetics*, vol. 25, no. 9, pp. 1269–1279, September 1995.

[10] E. Frazzoli, Z.-H. Mao, J.-H. Oh, and E. Feron, "Resolution of conflicts involving many aircraft via semidefinite programming," *Journal of Guidance, Control, and Dynamics*, vol. 24, no. 1, January-February 2001.

[11] G. Inalhan, D. Stipanovic, , and C. Tomlin, "Decentralized optimization, with application to multiple aircraft coordination," in *Proceedings of the 41st IEEE Conference on Decision and Control*, Las Vegas, December 2002.

[12] L. Pallottino, E. Feron, and A. Bicchi, "Conflict resolution problems for air traffic management systems solved with mixed integer programming," *IEEE Transactions on Intelligent Transportation Systems*, vol. 3, no. 1, pp. 3–11, March 2002.

[13] S. Boyd, S.-J. Kim, L. Vandenberghe, , and A. Hassibi, "A tutorial on geometric programming," *Optimization and Engineering*, vol. 8, no. 1, pp. 67–127, 2007.

[14] P. Menon and G. Sweriduk, "Optimal strategies for free-flight air traffic conflict resolution," *Journal of Guidance, Control, and Dynamics*, vol. 22, no. 2, pp. 202–211, 1999.

[15] A. Lecchini Visintini, W. Glover, J. Lygeros, and J. Maciejowski, "Monte carlo optimization strategies for air-traffic control," in *AIAA Guidance, Navigation and Control Conference and Exhibit*, San Francisco, California, 2005.

[16] E. Clarke, "Multi-part pricing of public goods," *Public Choice*, vol. 11, pp. 17–33, 1971.

[17] T. Groves, "Incentives in teams," *Econometrica*, vol. 41, pp. 617–663, 1973.

[18] J. Löfberg, "YALMIP: A toolbox for modeling and optimization in MATLAB," in *Proceedings of the CACSD Conference*, Taipei, Taiwan, 2004. [Online]. Available: http://control.ee.ethz.ch/~joloef/yalmip.php